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LUMINOSITY OF PLANETARY NEBULAE AND STELLAR TEMPERATURES

BY

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ABSTRACT

The present work was undertaken with the view of testing the theoretical mechanism underlying the luminosity of planetary nebulæ, and of devising methods for determining the temperature of the central star from the observed luminosity. A brief account of the main results has been given in a letter to "Nature", *121*, 790 (1928).

Spectrophotometry of the slitless spectrograms.—Slitless spectrograms were secured for three planetary nebulæ, with the ultraviolet spectrograph. On the same film on which the slitless spectrum of the nebula and its central star was taken, a series of comparison spectra of varying intensities was impressed with the same time of exposure as that of the nebula. The comparison spectra were obtained from a sensitometer having the daylight sky as a source, a real image of the sensitometer patches being formed at the slit of the spectrograph. The photographs yield the ratio of the total intensity of each monochromatic image of the envelope L_p to the intensity per frequency unit $\frac{\delta L_s}{\delta \nu}$ of the star spectrum at its centre, which has the same wavelength. The results of the observations are

represented by the quantities $A_\nu = L_p / \left(\nu \left(\frac{\delta L_s}{\delta \nu} \right) \right)$ determined for each monochromatic image. For the extreme

images $H\beta$, N_1, and N_2 an extrapolation of the star spectrum is necessary.

Theory of Nebular Luminosity.—The exciting star is replaced by a black body of surface temperature T and the luminosity assumed to be produced by two mechanisms. *Mechanism 1* is that of ionization of nebular atoms by ultraviolet starlight and subsequent recombination of the photo-electrons, and is applicable to the spectra of H, He_I, and He_{II} in particular. It was conceived of by D. H. Menzel and the author independently and elaborated for diffuse nebulæ by the author. *Mechanism 2* is that of electron excitation by photo-electrons freed by mechanism 1 before they recombine. It is applicable in particular to the nebulium resonance lines and was put forward by I. S. Bowen.

Result of temperature determinations and test of the theory.—Under certain approximate assumptions it is possible to derive formulæ for temperature determinations of the central star based on both mechanisms. The observed A_ν values for H, He_I, and He_{II} give independent determinations of the temperature T of the central star by mechanism 1, whereas the A_ν's for the nebulium lines give one determination of T by mechanism 2. The results of the various methods are the following:

N.G.C. 6543: 39,000° (H), 37,000° (Neb.).

N.G.C. 6572: 40,000° (H), 35,000°–42,000° (He₁), 38,000° (Neb.).

The values of A_ν for the third nebula N.G.C. 7009 are not trustworthy, but the He_{II} yields a temperature of approximately 70,000°, and the H and nebulium values are lower by about 20,000°.

The temperatures are only approximate and represent, strictly speaking, lower limits, but the agreement between the values derived by different methods is a test of the theory.

Temperature from difference in magnitude of star and nebula.—The photographic brightness of O stars is approximately proportional to the spectral intensity of the isophotic wavelength, about λ 4210, according to A. Brill, and the visual brightness of the nebula is mainly due to N_1 and N_2. A formula relating the difference in magnitude d of star and nebula, and the temperature T of the central star is derived, involving only one constant C which may be fixed by one nebula for which T by the nebulium method is known. Using the nebulæ N.G.C. 6543 and N.G.C. 6572 to fix the temperature scale, approximate temperatures are derived from the difference of the photographic magnitude of the star (Curtis, Hubble, and Van Maanen) and the visual magnitude of the nebula (Holetschek and Hopmann). From about 60,000° on the temperatures thus determined must be considered as lower limits. Temperatures all the way up to 100,000° are found. Of particular interest is the star in the Crab Nebula with a minimum temperature of 100,000° according to the theory of luminosity discussed.

A theoretical justification for the assumption that all the light in the Lyman series leaving the nebula is concentrated in the first line is given. It is based on Sugiura's values for the number of dispersion electrons in the Lyman series and the corresponding continuous spectrum obtained from the wave mechanics, and assumes besides that the absorption of ultraviolet starlight is fairly complete.

Difficulties raised by H. H. Plaskett are pointed out. The conclusion is reached that, though the approximate nature of some of the assumptions is realized, the theory discussed appears to present a fair interpretation of the luminosity data of planetary nebulæ.

INTRODUCTION

The purpose of the present investigation was to secure luminosity data for a few planetary nebulae. Apart from the purely observational interest of the problem, this was done with a double object in view: in the first place in order to investigate whether known physical theories would be capable of explaining the observed phenomena under the assumption that the central star is a black body, and in the second place, this being the case, to obtain the temperature of this star from luminosity data only. The discussion of the theoretical aspects of the problem to which different authors have contributed, will be taken up in Section 4, and the theory applied to the observations in Section 5. Under the assumption that the central star is a black body it will be possible to derive its temperature from luminosity data only by two entirely different methods and the approximate agreement of the two methods justifies the conclusion that the underlying physical assumptions are essentially correct, so that both a check of the theory has been possible and a fairly reliable temperature determination based on it has been carried out.

Attention will first be mainly confined to the observational side of the problem. As is well known, a typical planetary is a nebula of the gaseous type with a rather definite shape, usually appearing on the photographs as a disc or ring with a star approximately at its centre. A slitless spectrogram shows the spectrum of the star as a continuous streak and the spectrum of the gaseous envelope as a series of monochromatic pictures superimposed on it, so that the wavelength of each image is approximately the same as that of the continuous stellar spectrum at its centre.

For the purpose of a theory of luminosity it is evidently important to know the total intensity of each monochromatic image in terms of the total intensity per frequency unit of the adjoining star spectrum. My original idea was to take a series of photographs of the same nebula, varying the intensity by means of some polarizing device, and thus make the surface intensity of one of the monochromatic images in one picture equal to that of the same wavelength of the stellar spectrum in another picture. Mr. H. H. Plaskett, however, pointed out that such a method would not be feasible. He suggested to take a slitless spectrogram of the nebula with the ultraviolet spectrograph and afterwards impress on the same film a series of monochromatic comparison patches of known intensity ratios obtained by means of a suitable colour screen, which method could, e.g., be applied to the image of λ 4686 due to ionized helium.

This idea was further elaborated and it appeared that it appeared of great advantage to have, instead of the series of monochromatic comparison patches, a series of comparison spectra of known intensity ratios, impressed on the same film as the slitless nebular spectrum, since then it would be possible to find the ratio of the surface brightness of each nebular image to that of the adjoining star spectrum from one photograph. Such a series of comparison spectra might be produced by placing a sensitometer of sufficiently small size at the slit of the spectrograph. It is not easily possible to construct a sensitometer of such small dimensions, but instead of this one may take a sensitometer of ordinary size, and, by means of a plane and a concave mirror, form a small real image of the sensitometer patches at the slit of the spectrograph. It was first intended to use an incandescent lamp as the

source of light for the sensitometer, but this source has the disadvantage of being rather weak in the ultraviolet. For this reason daylight was tried as a source. The light of the sky, if reflected on an opal glass plate behind the sensitometer, did not produce any spectra at all on the film for an exposure of about one hour, hence a stronger source of illumination had to be looked for. A more powerful source was found in the light of the sky itself, reflected directly into the sensitometer by means of a mirror, so that the reflected sky itself formed a background of illumination which, on account of the small portion of the sky used, had a sufficiently uniform surface brightness.

The method of putting on the comparison spectra which was actually used may be seen from fig. 1, and was briefly as follows. In the daytime the spectrograph was taken down from the reflector and placed in the dark room clamped on a wooden base, and the slit was put back in the instrument. A sensitometer S had been placed in the wall of the dark room, so that it received its light through a window of the adjoining room, the light of the sky being reflected in it by means of a plane mirror M_1. Inside the dark room at b were therefore the six sensitometer patches of known intensity ratios. By means of a plane mirror M_2 with a central hole and a concave mirror M_3, a real image of these patches, one-fifteenth of their actual size, was formed at the slit Sl of the spectrograph. The film, which had already been exposed to the nebula, was then placed inside the film holder and the latter attached to the spectrograph, taking care that the exposed part of the film was sufficiently far from the axis, which could be done by moving the film holder by means of a micrometer screw. The exposure of the comparison spectra then took place with the same time of exposure as that originally used for the nebula. Details are given in Section 1.

The time for the observational work was rather limited, since it had to be carried out during the summer vacation of 1927. At first most of the time was spent in working out the device for putting on the sensitometer spectra and in construction of the apparatus needed for this purpose. In some respects this was not a great loss, since conditions for observational work soon became very unfavourable on account of the smoke of forest fires. The first photographs of nebulae obtained, six in all, were inferior, mainly through imperfection in the method of guiding, which was at first done on the nebula with the 7-inch finder, but they gave the necessary experience in handling the instrument and served as an orientation. It was not until the 7th of September that the difficulties in guiding were definitely overcome by introducing a method discussed in Section 2, which is in principle as follows. One of the slit jaws of the spectrograph is replaced by a polished brass door with a cross engraved on it. For one minute the door is shut and during that time the image of the nebula is placed at the centre of the cross. The door is then opened and a 4-minute exposure takes place, after which period the door is shut again for one minute and the nebula is brought back to the centre. Thus an intermittent exposure in periods of four minutes is given, and though it may seem an unnecessary precaution, the exposure of the comparison spectra put on later was also made intermittent in the same way. Through the courtesy of members of the staff who were willing to shift or give up some of their observational nights, I was enabled to work during the nights of September 13, 14, and 16. Four photographs of good quality were secured of N.G.C. 6543 and N.G.C. 6572, which will be fully discussed. Two other photographs

of N.G.C. 7009 were strongly fogged by moonlight and the results of these are not trustworthy. In so far as conclusions can be drawn from them, they will be briefly mentioned.

The measurement of the films and their provisional interpretation took place in the winter of 1927-8, during a quarter's leave of absence from the University of Washington. The calibration of the wavelength and the measurements of density with the Hartmann wedge microphotometer were carried out in Victoria, and the further elaboration of the results at the California Institute and the Mount Wilson Observatory in Pasadena, where also a few measurements with the thermocouple were carried out in order to furnish a check of the wedge measurements. In the meantime a completely new light had been thrown on the results of nebular investigations by Dr. Bowen's discovery of the origin of the most prominent nebulium lines, and his work on the structure of planetary nebulae was in process of publication. Through his kindness I had access to the manuscript of the latter work and was enabled to discuss the different features of the present investigation with him.

If formerly there had been only one way of obtaining the stellar temperature from luminosity data of nebulae, namely by means of the theoretical mechanism of ionization and recombination, it is now possible to obtain an independent temperature determination from the strength of the nebulium lines by means of a different mechanism, put forward by Bowen, to be discussed in Section 4.1. It is the check between these two entirely different methods which enables us to judge in how far the theoretical assumptions are justified. A provisional account of the present work has been given in a letter to Nature[1].

The first three sections of the paper are concerned with the observational part. Section 4 and those following it, dealing with the theory and the discussion of the observations, may be read without reference to the previous sections, apart from some minor details.

SECTION 1.—METHOD OF IMPRESSING THE SENSITOMETER SPECTRA ON THE FILM

1.1. *Arrangement of the Apparatus. Construction and dimensions of the parts.*—The arrangement of the apparatus for putting the sensitometer spectra on the film, briefly referred to in the introduction, is shown in the schematic drawing of fig. 1 and in the photograph of Plate VIII, fig. 4. In fig. 1 the light of the sky, which enters through a window in the room adjoining the dark room, is reflected by the plane mirror M_1 into the six holes of the sensitometer S at a. The other side of the sensitometer at b inside the dark room is covered by a piece of opal glass on which six rectangular spots of light, each of them illuminated by one of the circular holes at a, are faintly visible for an observer in the dark room. The light from the six rectangular patches of the sensitometer passes through the tube T in the frame of the spectrograph, is first reflected by the plane mirror with circular hole M_2, and subsequently by the concave mirror M_3, when it passes through the hole in M_2 and comes to a focus at the slit Sl of the spectrograph. The images of the sensitometer patches which are one-fifteenth of the size of the patches may be observed on the edges of the slit by means of the guiding telescope, not shown in the figure. Since the magnification is one-fifteenth and the focal length of the concave
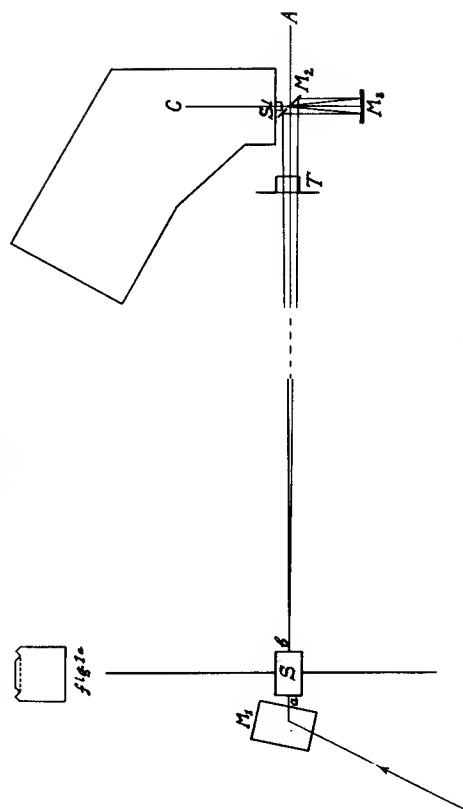
¹ Nature 121, 790 (1928).

Fig. 1.—Arrangement of the apparatus for putting on the sensitometer spectra. M_1 plane mirror. S sensitometer. T tube in frame of spectrograph. M_2 plane mirror with central hole. M_3 concave mirror. Sl slit of spectrograph. A real image of the sensitometer patches at b is formed at Sl.

Fig. 1a.—In upper left-hand corner. Piece of brass for horizontal partitions in sensitometer.

mirror M_3 is 16·4 cm., it follows that the distance of M_3 to the slit is 17·5 cm., and the optical distance of the object b to M_3 is 262 cm. The plane mirror M_2 was fixed to the wooden box of the spectrograph; it made an angle of approximtaely 45° with the optical axis and its centre was 1·7 cm. in front of the slit. From this it follows that the distance of the sensitometer cover b to the centre of the plane mirror M_2 had to be 246 cm.

The photograph of Plate VIII, fig. 4 was taken outside the dark room, to give an idea of the arrangement of the apparatus. The black box at the left-hand side of the picture is the sensitometer. Its actual distance from the spectrograph was greater than shown in the photograph. By means of two screws it was fixed in the sill of a window communicating with the dark room, and the rest of the window was covered up by a shutter. The plane mirror by which the daylight was reflected into the sensitometer is not shown. The spectrograph is mounted on a wooden shelf placed on the bench in the dark room. The ring by which the spectrograph is ordinarily mounted on the upper end of the tube of the reflector by means of four screws is clearly shown. The two lower screws served to fix the ring of the spectrograph to the shelf, as is better seen in fig. 5, while the other side of the spectrograph was merely dropped in place on a wooden base in which it fitted. The guiding telescope through which the image of the patches on the slit could be observed was also attached to the instrument. The light from the sensitometer passed through a circular hole of 5 cm. diameter in the frame of the spectrograph (tube T of fig. 1) before it came to the plane mirror M_2; however this is better shown in fig. 5.

Plate VIII, fig. 5, shows the mounting of the plane and the concave mirror, and the prism through which the light from the slit is reflected into the guiding telescope. This prism marked by an arrow has to be removed before the actual exposure takes place,

since it is partly in the way of the light rays. In this connection it may be remarked that special care was taken to insure that the rays of light were unobstructed and that the cone of light incident on the slit amply filled the collimator tube, apart, of course, from the necessary central hole in the cone due to the presence of the hole in the plane mirror. Also it is necessary that, on the passage of the rays through the different parts of the spectrograph, the dark cone where the rays are cut out by the hole is always completely inside the light cone, and for rays emerging from the slit outside the axis also, for if it were not, an error in the intensity of some of the comparison spectra away from the axis would be introduced. That all these conditions were fulfilled could easily be verified on a drawing of the apparatus, making use of the fact that the focal ratio of the collimator tube was 5 : 1.

To the right in fig. 5 the wooden box enclosing the spectrograph is seen, to which the two triangular supports for the plane mirror are attached. The plane mirror is 7·2 cm. square and made of plate glass 3 mm. thick; at the centre is a circular hole of 15 mm. diameter with bevelled edge. The mirror can be slipped into its place between the two triangular supports and was removed when the spectrograph was attached to the reflector. Toward the right of the central hole the micrometer screw regulating the width of the slit is seen. A concave mirror of good quality was obtained by a very simple device due to Mr. P. M. Higgs of the Department of Physics of the University of Washington, namely, by silvering one of the surfaces of a concave lens. The lens which served as a mirror was clamped in a brass strip mounted on a wooden support by means of three adjusting screws, as seen on the left of fig. 5. The back of the wooden support is also visible in fig. 4. The three mirrors M_1, M_2, and M_3 were silvered by the Brashear process[2] and afterwards polished.

In fig. 5 the collimator of the spectrograph is inside the wooden box at the right. Its optical axis is in the direction of the line joining the centre of the concave mirror with the centre of the hole in the plane mirror. The light from the sensitometer is incident perpendicular to this axis. It enters through a circular tube in the frame, clearly seen in fig. 5, and indicated by the letter T in fig. 1. In order to give the comparison spectra the intermittent exposure referred to in the introduction, a shutter, not shown in any of the figures, was placed in front of this tube T. During the exposures the shutter was intermittently closed for one minute and opened for four minutes.

1.2. *The sensitometer.*—The sensitometer is shown in Plate VIII, fig. 6. One of the sides of the box is removed in order to show the construction. The back cover of the sensitometer, where the light enters, consists of a brass plate with six circular holes of varying size. The front plate of the sensitometer was made of fairly thick paper, as is used for calling cards, and has six rectangular holes, the centres of which are 1 cm. apart. It is covered by a piece of opal glass.* Out of the same paper three diaphragm plates were made, in each of which six openings were cut. The diaphragm plates are placed inside the sensitometer parallel to back and front plates, so that each hole serves as a diaphragm for the beam passing from one circular hole in the back plate to the corre-

[2] H. D. Curtis, Publications of the Astronomical Society of the Pacific 23, 20 (1911).
* The two additional square holes seen in the photograph were added with the intention of using them for putting on a line spectrum to fix the wavelength of the sensitometer spectra, but since the H and K lines in absorption were clearly visible in the sensitometer spectra no such comparison spectrum was needed.

sponding rectangular hole in the front plate. The back and front plates, as well as the three diaphragm plates, were now slipped into a U-shaped wooden box, the cover of which could be removed, as shown in the figure. The horizontal partitions visible in the photograph were then constructed as follows: Twenty-eight pieces of thin brass plate were cut in the shape of fig. 1a. The central part was folded over rectangularly along the dotted line, so that the two triangular points were sticking out. The pieces were then put into their places by pressing the triangular points into the wood, thus keeping the paper diaphragms and front plate in place, as well as forming the six horizontal tubes through which the light passes. All parts of the sensitometer were then covered with a dull black paint. By looking through the square holes in front, it was made sure that the diaphragms actually cut out the stray light reflected from the walls and the partitions. Where this was not yet the case, namely for the two lower openings, a few additional diaphragms were put in, as shown in the figure. When the side cover was put on the sensitometer box, it was covered on the inside with a piece of black cloth so as to make a tight fit. The dimensions of front and back plates were 9 by 2 cm. and their distance apart was 10 cm.

The plane mirror M_1 of fig. 1 which reflected the light of the sky into the sensitometer consisted of a piece of very thick plate glass silvered by the same process as the other two mirrors. It was placed on a wooden shelf which could be adjusted into the desired position. This mirror is not shown in any of the photographs.

When the apparatus was not in use, all mirrors were always removed or covered up.

The surface brightness of each of the six sensitometer patches is proportional to the area of the corresponding hole in the back plate. It was intended to have the sensitometer cover a range of about five stellar magnitudes, so that the ratio in brightness of two consecutive patches would correspond to approximately one magnitude. This is the case when the ratio of two consecutive diameters is approximately 1·584. The Director, Dr. J. S. Plaskett, selected the following diameters of the holes as practicable: 0·025, 0·040, 0·063, 0·100, 0·158, 0·250 inch, so that the last five could be made by drills of standard size (Nos. 60, 52, 39, 21, and ¼''), and he had the plate accordingly prepared. The centres of the holes are 1 cm. apart and the edges are bevelled. Like all parts of the sensitometer, the plate was painted a dull black.

Later on, the diameters of the holes were measured accurately with the micrometer-microscope. The readings were taken both when measuring in the direction of the line joining the holes and perpendicular to it. The plate was illuminated mainly from above, with a faint illumination from below. The measurements were taken with the smaller opening of the bevelled holes upward. Measurements with the smaller opening downward were also taken for the smallest three holes, but they served only as a check and were not used. The result is given in Table 1.

TABLE 1.—DIAMETERS AND AREAS OF HOLES IN BACK PLATE OF SENSITOMETER

Hole	1	2	3	4	5	6
Diameter, mm.	0.572 ± .006	0.992 ± .004	1.513 ± .003	2.542 ± .003	4.041 ± .008	6.334 ± .002
Area	0.817 ± .017	2.455 ± .020	5.70 ± .02	16.11 ± .04	40.20 ± .16	100.0 ± .0
Log. Area	9.912 ± .010	0.390 ± .004	0.756 ± .002	1.207 ± .001	1.604 ± .002	2.000 ± .000

The area is expressed in such a way that the area of the largest hole is equal to 100. The error given is the deviation from the average of the two values. For the smallest hole it is as much as 2 per cent, but for the largest four holes, which were practically the only ones used, it is only 0.4 per cent, or less, which is quite insignificant for the present purpose.

1.3. *Adjustments.*—The instrument used was the Hilger Ultraviolet Spectrograph G 7, with two ultraviolet crown prisms. The spectrum is taken on a photographic film placed in the curved film holder shown on the right of fig. 6, so as to compensate for the strong curvature of the field. The focussing is brought about by moving the carrier of the film holder in- and outward and by tilting it. For focussing the spectrum a copper arc was used. The prisms were first adjusted so that the line H_δ would be near the centre of the film, and the proper focussing determined by a series of trial exposures of varying focus and tilt.

The radius of curvature of the concave mirror M_3 was determined in the usual way, by having an object (paper scale) and its real image coincide without parallax. The radius was determined as 32.8 cm., hence the focal length 16.4 cm.

The parts of the apparatus for putting on the sensitometer patches in the dark room, described in Section 1.1, needed the following adjustments (see fig. 1), not including as yet the concave mirror M_3:

1. The centre of the hole in the plane mirror M_2 should be on the optical axis $Sl\ C$ of the collimator tube.

2. The line $b\ M_2$ connecting the centre of the sensitometer with the centre of the hole in the plane mirror should be perpendicular to this optical axis. From a drawing this line was found to be the case when this line $b\ M_2$ passes at a distance of 6 mm. from the centre of the tube T, and this excentrical axis, seen as $b\ M_2$ in fig. 1, will be referred to as the optical axis of tube T. It could be marked by placing a semi-circular piece of cardboard with a mark on it in T.

3. The plane mirror M_2 should make an angle of 45° with both $b\ M_2$ and the optical axis $Sl\ C$ of the collimator. This condition is evidently satisfied if, looking along the optical axis of the collimator into the plane mirror M_2, the image of the sensitometer is seen symmetrically with respect to the hole in M_2, provided that conditions 1 and 2 are satisfied.

The shelf on which the apparatus was mounted could always be put in its proper

position on the bench in the dark room by pressing its back against a wooden rail and having its side coincide with a line drawn on the bench, and it was kept in this position by putting heavy weights on it. One side of the spectroscope was fixed to the shelf by means of two screws and the other end dropped into the wooden support (figs. 4 and 5).

On Mr. H. H. Plaskett's suggestion, the optical axis of the collimator was located by removing the slit and observing an image of some external object in the collimator lens. For this object a mark of white paper on a wooden support was used. The optical axis is located when the paper mark, its image in the collimator lens, and the circular opening of the collimator are seen in one line; the axis is parallel to the shelf when this is the case for different positions of the support with the paper mark. By using this criterion, the support of the spectroscope was so corrected as to make the optical axis parallel to the shelf. This axis could always be located by means of the paper mark, and condition 1 could be made sure of.

To satisfy condition 2, it was necessary that the sensitometer, the optical axis of the tube T (marked in the manner described), and the centre of the hole in M_2 were in one line. This was made sure of by looking through the hole of the plane mirror in the direction $A\ b$.

To satisfy condition 3, it was necessary to change the position of the plane mirror somewhat by putting thin brass rings around some of the screws fixing its supports to the box, between the supports and the box. Knowing the optical axis by looking along paper mark and hole of the mirror, the adjustment was checked in the manner described under 3.

After the adjustments 1, 2, and 3 were made, the concave mirror M_3 was adjusted so as to have the images of the sensitometer patches in focus at the required position on the slit. This was done by means of the three adjusting screws (Section 1.1 and fig. 5). The centre of the slit was marked by a dot of white paint on a small piece of black paper stuck to one of the jaws, and the three screws were turned until the six images, viewed through the guiding telescope, were in focus and distributed symmetrically with respect to this centre.*

Once the adjustments 1, 2, and 3 had been made, a simpler verification was carried out in the future.

Conditions 1 and 3 remain practically satisfied, since the support of the mirror is fixed to the box, and it was made sure during the tests that slipping in and out of the mirror did not alter these adjustments. Also these two conditions can only be verified when the concave mirror and its support are completely removed, which is very undesirable.

When therefore a photograph of the comparison spectra was taken in the dark room, only adjustment 2 was checked by verifying that the hole in M_2, the optical axis of T and the centre of the sensitometer were lined up properly. The six images on the slit were then brought in focus and in their proper position by adjusting the concave mirror M_3, and subsequently the exposure took place.

1.4. *Exposures and development conditions.*—The photographs were taken on East-

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* By a somewhat laborious method, the mirror M_3 was also adjusted independently on the optical axis of the collimator at the required distance. The images on the slit were then found approximately in focus and in the required position, which served as a check of the other three adjustments.

man Portrait film, par speed, 4¼ by 6¼ inches, cut into sizes of 6·7 by 4·8 cm. The exposures varied from 45 minutes to 2½ hours and were intermittent, consisting of intervals of 5 minutes, in which the exposure was first shut off for one minute and then took place for 4 minutes. It is essential in the method that the time of exposure of the nebula be the same as that of the sensitometer spectra. The latter was made intermittent in the same way as for the nebula by means of the shutter on tube T referred to under 1.1.

After the film had been exposed to the nebula, it was exposed to the sensitometer spectra during a subsequent day at a convenient time, and then immediately developed. Reproductions of the results are seen in Plate IX, figs. 7a to 7f, showing the slitless spectrogram of the nebula with the sensitometer spectra. In every case there were six sensitometer spectra in all, but one or more of the faintest were not sufficiently intense to register on the film. The range of blackening offered by the sensitometer spectra was sufficient to fix an intensity scale for each monochromatic image and adjoining stellar spectrum, except for the images of N_1 and N_2, for which in most cases an extrapolation had to be applied, which makes the result less accurate.

For the development of the films, X-ray developer was used, the same as in H. H. Plaskett's photometric work.[3] The time of development was 4⅓ or 4½ minutes, the temperature about 23°C. It was originally intended to duplicate the conditions of development closely, so that the slope of the characteristic curve on one film could be used on another film, but this was not possible and it is not of much importance for the method used.

After the film was placed in the development tray, the developer was quickly poured on, and the time at which this took place was considered as the beginning of the development. The film then soon had a tendency to curl up and had to be kept under the developer by putting the thermometer on it. Though the thermometer was moved to different places as much as possible and care was taken not to touch the exposed part of the film with it, this may have introduced errors, since the flow of the fresh developer across the film is influenced by it. At the moment when the development should be stopped, the developer was quickly poured off, and the film placed under a jet of water, where it remained until it was clear. Fresh developer was used every time.

We are of the opinion that one source of error is due to the rather crude method of developing, which must result in somewhat different development conditions for different portions of the film. In the future it is desirable to give more attention to this matter, but as it was time did not permit it.

SECTION 2.—THE NEBULAE OBSERVED.   METHOD OF GUIDING.

2.1. *The nebulae observed.*—In the observational part of the work, which was entirely new to me, I had the very active help of the members of the staff, especially of Mr. H. H. Plaskett, and relied to a large extent on their guidance.

A number of suitable nebulae were selected for an observational program. Among the nebulae of the proper right ascension (16 h. to 23½ h.) only those should be considered as having radii of less than 15" (so that the images $H\beta$ and Fowler 4686 would be clearly separated) and a sufficient surface brightness (Curtis' relative exposure less than 5).

³ H. H. Plaskett. Publications of the Dominion Astrophysical Observatory, IV, 141 (1923).

Also the central star should not be too faint, 13^m or brighter. In Hubble's list[4] there are altogether ten nebulae of the proper right ascension satisfying these conditions, and the following were selected for an observational program: I.C. 4593, N.G.C. 6543, 6572, 6891, 7009, and 7662. The magnitude of the central star, in the succession given, is: 12, 11, 11, 12, 12, 13, whereas the total photographic luminosity, as Mr. H. H. Plaskett informed me, is brighter by 1, 2, 3, 0, 5, 5 magnitudes, thus they offer a range of various types.

TABLE 2.—THE NEBULAE OBSERVED

Object	m_*	m_t	diff.	A	e	Film No.
N.G.C. 6543	11	9	2	8"	0·1	7, 9, 12
6572	11	8	3	3"	0·05	10
7009	12	7	5	12"	0·1	6, 8

Unfortunately it was not possible to carry out this program in full, and only three of the six nebulae were observed. They are given in Table 2. The second column gives the photographic magnitude of the star m_* according to Van Maanen or Hubble[4], the third column the total magnitude m_t of the nebula including the star which value was furnished by Mr. H. H. Plaskett from unpublished work. The fourth column gives the difference $m_* - m_t$, the fifth column the radius A of its brightest portion from Hubble's list, the sixth column Curtis' relative exposure $e*$, and the last column the numbers by which the films are indicated. The spectral type of the star is known only for N.G.C. 6572 and is $W R1$, as Mr. Plaskett informed me. According to Wright[5] all three nebulae show a continuous spectrum at the head of the Balmer series, which is strong for N.G.C. 6543 and N.G.C. 7009, whereas the nucleus of N.G.C. 6543 has hazy emission lines or bands, many of which correspond to the so-called Wolf-Rayet radiation. Though the three nebulae should all be favourable for getting the intensity of the continuous spectrum at the head of the Balmer series, the measurements in the present work did not allow an estimate of its intensity. As regards the difference of stellar magnitude and total magnitude, it would have been of interest to include a case of a faint nebular envelope where therefore this difference is small, as e.g. N.G.C. 6891. Otherwise these differences, 2, 3, and 5, are fairly representative, since in the ten suitable nebulae the differences range from 0 to 5.

2.2. *The method of guiding.*—As was mentioned in the introduction, considerable difficulty was experienced in finding a proper method of guiding. There is no doubt that the procedure of guiding on the image of a neighbouring star, such as was used by Wright at the Lick Observatory in securing his slitless spectrograms, is preferable. However, at the Dominion Astrophysical Observatory no such arrangement was available. The guiding therefore took place on the nebula, at first through the 7-inch finder with lighted movable crosswires, but in several trials this method appeared not very satisfactory.

⁴ E. P. Hubble. Ap. J. 56, 400 (1922), Table III.
* This relative exposure e is defined as the time of exposure required to give the brightest part of the nebular envelope on Seed 27 or Seed 23 plates a certain density, divided by the time of exposure yielding the same density for a selected portion of the Orion Nebula. (Lick Observatory Publications, 18, 59, 1918).
⁵ Lick Observatory Publications, 13, 193 (1918).

The method of "door guiding" at the upper end of the tube, mentioned in the introduction, was then worked out. It has the advantage that all the light of the 72-inch reflector is available for the image on which the guiding takes place and that this image, as seen in the guiding telescope is quite large. The fixed jaw of the slit had to be replaced by a polished brass door which fitted to the movable jaw, after the latter had been moved away from the centre as far as possible. A device originally used at the Observatory for putting on comparison spectra could be adapted to this purpose. It consisted of a hinge carrying two prisms which, turning about the hinge, could be placed in front of the slit or removed from this position. By replacing the set of two prisms by a piece of polished brass on which a cross was scratched with a needle, the desired door was constructed.

The slit provided with the door is shown in Plate VIII, fig. 6, on the right, above the film holder. In order to bring out the details, the slit has been removed from the spectrograph and mounted on a piece of wood. The brass door is automatically shut by means of the two rubber bands on the right, it is opened by pulling a cord seen towards the left. Actually, when the slit was on the spectrograph attached to the tube, the cord was beside the guiding telescope. It could be pulled, and by putting it around a knob the door then remained open. On the other hand, when the cord was slackened, the door remained closed. By trials it was made sure that the position of the cross on the door was always the same when the latter was shut.

The actual guiding at the upper end of the tube was carried out as follows. After the nebular image was brought on the brass door, which was closed, the film holder was opened. One minute was spent in bringing the centre of the nebula as accurately as possible to the centre of the cross by pressing the control buttons changing the position of the tube. At the end of this minute, the door was suddenly opened. It remained open for four minutes, at the end of which interval the door was shut again by releasing the cord. Then the re-adjustment of the nebula on the cross took place, for which one minute was available. At the end of this minute the door was opened again, and so on. For looking at the watch giving the time for opening and shutting, a red flashlight was used. Shortly before the end of the four-minute interval, a warning signal was given by a development timer, wound up at the beginning of this interval.

Besides the inaccuracy in centering the picture, the error in the guiding will be due to the drift in between two subsequent adjustments. Once however one knows approximately how many seconds one has to press the right ascension and declination buttons in order to bring the nebula back, one may decrease this error by pressing the buttons half of this time, after two minutes. This correction, suggested by Mr. Plaskett, was often applied, when the drift became appreciable.

By applying this method of guiding, pictures quite suitable for measurement were obtained. However, it would appear that, when the nebula is large and faint, the guiding of the central star is not so good.

SECTION 3.—CALIBRATIONS AND MEASUREMENTS OF INTENSITY RATIOS.
RESULTS OF THE MEASUREMENTS.

3.1. Calibration of the wavelength scale.—The nebula N.G.C. 6572 (fig. 7d) shows sharp images which do not overlap, with the

exception of the nebulium images N_1 and N_2, $\lambda 5007$, 4959, due to O_{III}. At the suggestion of Mr. H. H. Plaskett, the slitless spectrograms of this nebula were used for calibrating the wavelength scale. Besides the film No. 10, actually used for intensity measurements, another film, No. 2, secured in preliminary tests, was at our disposal. On the latter film the image $\lambda 3726$ was too hazy and therefore discarded. The scale reading n in mm. corresponding to the different wavelengths λ was determined on a measuring machine, first measuring the pictures in one direction, and then taking another set of readings in the opposite direction. Making use of the readings for H_β, H_γ, and the nebulium image $\lambda 3726$, the relation between n and λ was expressed in the form of Hartmann's formula

$$n = 59 \cdot 069 - \frac{60037}{\lambda - 1808 \cdot 8} \quad (I),$$

where only two decimal places in n should be retained. The scale is considered such that H_γ is at $n = 35 \cdot 00$ mm. A comparison between calculated and observed reading is given in Table 3.

TABLE 3.—SCALE READING n IN mm. FOR DIFFERENT WAVELENGTHS λ IN Å

Line	N_4 (O_{III})	N_4	H_ζ	N_4	H_ϵ	H_δ	H_γ	H_β
λ	3726	3869	3889	3967	3970	4102	4340	4861
Calc. n	27·29	29·49	29·77	30·84	30·87	32·49	35·00	39·10
Obs. n	27·29	29·45	......	30·82		32·47	35·00	39·10

Line	N_1 (O_{III})	N_2 (O_{III})
λ	4959	5007
Calc. n	39·72	40·01
Obs. n	39·67	40·01

Comparison of the observed with the calculated values of n in the table shows that an error of a few units in the second decimal of n may occur, which, for the present purpose, is sufficiently accurate. The lines H, $\lambda 3969$, and K, $\lambda 3934$, of calcium occurring in the sensitometer spectra correspond to scale readings $n = 30 \cdot 85$ and $30 \cdot 39$ mm. by (I).

3.2. Determination of the intensity scale for each wavelength with the microphotometer.—In Plate IX, figures 7a to 7f, reproductions of the films measured under the microphotometer are shown. Each photograph shows the star spectrum as a continuous streak on which the monochromatic nebular images are superimposed, and, below this, a number of sensitometer spectra, in the strongest of which the H and K lines of calcium are visible. The scale of intensity for each wavelength is now fixed as follows:— The film, between two pieces of glass, is placed under the microscope of the microphotometer, where it can be displaced by a horizontal and a vertical micrometer screw. By moving the horizontal micrometer screw backward and forward the sensitometer spectra are lined up horizontally in the field, taking care at the same time that the micrometer readings of the H and K lines in the strongest spectrum are as close as possible to the calculated values of n, 30·85 and 30·39 mm. After a few trials, the positions of H and K when measured are sufficiently close to the desired values, and each scale reading corresponds to the wavelength as given by the Hartmann formula (I). The densities of blackening D in the sensitometer spectra are now determined for different scale readings n of the horizontal micrometer screw, that is for different wavelengths, as follows:—For a fixed n
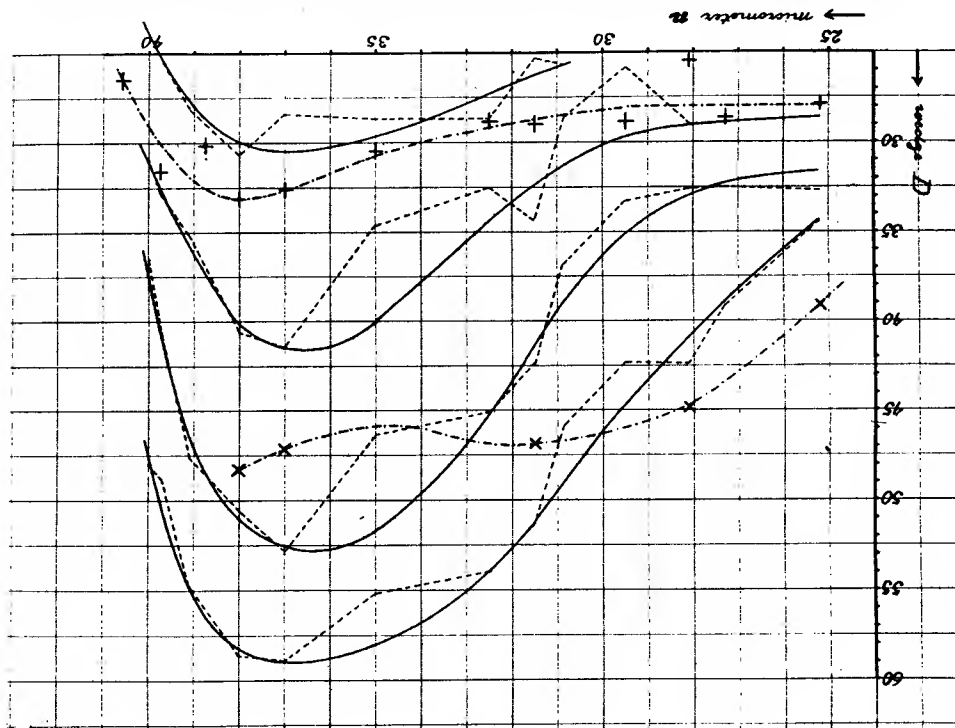
a setting is made on the central part of the strongest sensitometer spectrum, and the density there determined. By moving the vertical micrometer screw, leaving n the same, the densities of the central part of the other sensitometer spectra are measured in the same way. This is repeated for different values of n.

Fig. 2 shows the results of the readings for film No. 9, N.G.C. 6543, with the Hartmann wedge microphotometer. The scale reading n is plotted horizontally, the wedge reading D, representing the density of blackening in arbitrary measure, is plotted vertically. For each sensitometer spectrum the broken dotted line connects the individual determinations, the highest of them representing the readings of the strongest sensitometer spectrum. It is seen that the set of points for each of the spectra does not represent a smooth curve. This is partly due to observational errors, but mainly to the strong absorption lines in the ultraviolet solar spectrum, since the deviations from a smooth curve are mainly systematic.* A freehand curve is therefore drawn through each of the broken dotted lines, taking care that the systematic deviations representing absorptions are all below the corresponding curves. This seemed the best procedure to correct for accidental errors, no further correction being applied from now on.

For a fixed wavelength λ, and corresponding scale reading n, the brightness of each sensitometer spectrum is proportional to the area of the hole given in Table 1. The four sensitometer spectra in fig. 2 form therefore a scale of intensities 5·70, 16·11, 40·2, and 100, of which the logarithms are 0·756, 1·207, 1·604, and 2·00. On the other hand, the drawn curves in fig. 2 represent for each n the wedge readings D corresponding to this arbitrary but true intensity scale. Thus for a given λ and corresponding n (calc. n of Table 3) the wedge readings are taken from fig. 2 and plotted against the logarithms of the intensities. This gives the characteristic curves fig. 3a and fig. 3b for the different wavelengths in the nebular spectrum and for $\lambda = 4701$, the longest wavelength for which the continuous star spectrum could be measured.

If now for two images of the same wavelength, that is the nebular image and the adjoining star spectrum, the densities D are determined as wedge readings, the surface intensities I may be taken from fig. 3a or fig. 3b. The unit in which I is expressed is 1/100 of the surface brightness in the strongest sensitometer spectrum and is therefore not known; however, the ratio of the two surface intensities I is known, since it does not depend on the unit.

3.3. *Determination of the quantities A_ν.*—As has been stated in the introduction, the purpose of the method is to determine the ratio of the total intensity L_p of each monochromatic picture of frequency ν to intensity per frequency unit $\frac{\delta L_s}{\delta \nu}$ of the continuous star spectrum at the same frequency. Dividing this ratio by ν we obtain for each picture the observational quantity A_ν,

$$A_\nu = \frac{L_p}{\nu \dfrac{\delta L_s}{\delta \nu}} \tag{II},$$

which will be used instead, since it has the advantage that it is dimensionless.

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* If the slit for the sensitometer spectrum had been narrow, it would have represented itself as a continuous spectrum crossed by absorption lines. With the wide slit used, these absorption lines are largely fuzzed out and the course of the spectrum is more or less continuous, except e.g. at $n=35\cdot00$ where a strong absorption band occurs, which explains the largest systematic dip in fig. 2 at $n=35\cdot00$.
775—2½

Fig. 2.—Measurements with the Hartmann wedge microphotometer; N.G.C. 6543, film No. 9 (reproduced in Plate IX fig. 7b). Scale reading n of micrometer, convertible into wavelength by Table 3 or formula (1) is plotted against wedge reading of blackening D. The D for the sensitometer spectra is represented by the drawn lines, for the star spectrum by the upper chain line, and for the fog by the lower chain line.

The quantities L_p and $\nu \frac{\delta L_s}{\delta \nu}$ are obtained by the photometry of nebular images and continuous star spectrum. To this end the star spectrum, seen in the upper parts of figures 7a to 7f, is lined up horizontally under the microscope of the microphotometer by making sure that the intersection of the cross wires used for the setting remains at the centre of the spectrum when the horizontal micrometer screw is moved. At the same time care is taken that the centre of the nebular picture H_γ is close to $n=35.00$. The position of the centre of this image is then determined accurately. If it is $n=35.00$ the scale reading λ for any n of the continuous star spectrum is known from (I) or Table 3, assuming the star to be central. If it is $n=35.00+ \triangle n$, the scale readings first have to be diminished by $\triangle n$ before calculating the wavelength.

The average surface intensity I_p of each monochromatic picture is now determined by a few measurements of D at different settings on the image. The values of D are first reduced to intensities I from the characteristic curve (for film No. 9, fig. 3a or fig. 3b), which intensities averaged give I_p. By measuring the two diameters of the picture (considered elliptical) with the micrometer microscope, its area σ in mm² is known. The total intensity of the picture L_p is then

$$L_p = I_p\,\sigma \qquad (III),$$

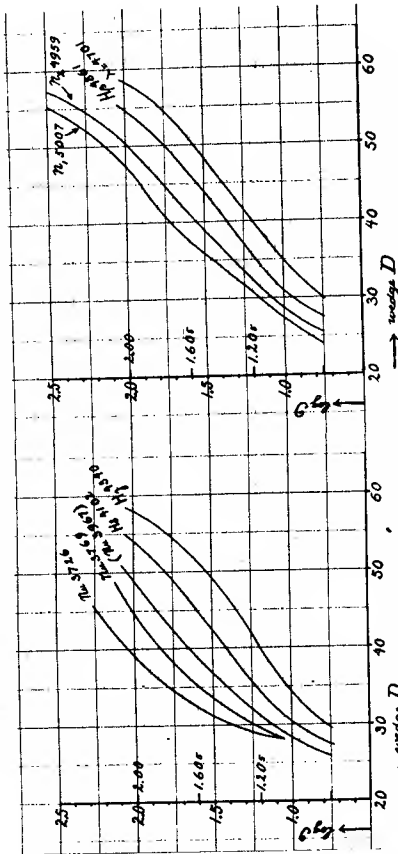expressed in the arbitrary but true intensity scale for the frequency ν of the image.

Fig. 3a and 3b.—Characteristic curves obtained from fig. 2. Wedge reading D is plotted against the logarithm of the intensity I of the sensitometer. The wavelength in Å for each characteristic curve is given in the figure.

The density of the central portion of the continuous star spectrum is now determined at different positions in between the nebular images, where it is free from the influence of the latter, but it is wanted at the nebular frequencies, that is at the centre of the star images. For film No. 9, N.G.C. 6543, the wedge readings D for the centre of the star spectrum are marked in fig. 2 by crosses (X). The chain line drawn through the crosses as a smooth curve provides a reasonable interpolation by which the wedge reading for

any intermediate point free from the influence of the monochromatic images is known. This line gives the desired density D_s for the star spectrum for each nebular frequency, except for the frequencies of the strongest nebular images H_β, N_2, and N_1, since, due to the falling off of the photographic sensitivity, the stellar spectrum beyond N_1 is no more visible.

For a nebular frequency ν, and corresponding scale reading n, the density D_s is taken from the chain line through the crosses (X) and, by means of the characteristic curve (fig. 3a) reduced to the surface brightness I_s at the centre of the star spectrum. However, for the images H_β, N_2, and N_1, the interpolation is not possible, and instead of this the value of I_s is taken for the point of the stellar spectrum measured nearest to H_β ($n=38.00$, λ 4701 in fig. 2). This value of I_s may be somewhat too high, but presumably by not more than 25 per cent.*

In measuring the films, the unit of length employed is the mm, and I_s may therefore be conceived of as surface intensity per mm², in accordance with the definition of surface brightness for the nebular images. When now, as an approximation, the surface intensity is considered constant and equal to its maximum value I_s for the width δ of the stellar spectrum as measured in mm. under the microscope, the quantity $I_s\delta$ represents the intensity of the star spectrum up to a certain frequency ν, or beyond the scale reading n.**

$$-\frac{\delta L_s}{\delta n} = I_s\delta \qquad (IV).$$

We have further, since $\nu = \frac{c}{\lambda}$ and the scale reading n is a function of ν,

$$\nu \frac{\delta L_s}{\delta \nu} = -\lambda \frac{\delta L_s}{\delta \lambda} = -\frac{\delta L_s}{\delta n}\,\lambda\,\frac{\delta n}{\delta \lambda} \qquad (V).$$

From (II), (III), (IV), and (V) follows

$$A_\nu = \frac{I_p}{I_s\,\delta}\,\frac{\sigma}{\lambda}\,\frac{1}{\lambda}\,\frac{\delta \lambda}{\delta n} \qquad (VI).$$

Moreover, by differentiation of the Hartmann formula (I)

$$\frac{1}{\lambda}\frac{\delta \lambda}{\delta n} = \frac{1}{\lambda}\,\frac{(\lambda - 1808.8)^2}{60937} \qquad (VIa).$$

For a nebular image of given λ the factor $\frac{1}{\lambda}\frac{\delta \lambda}{\delta n}$ in (VI) is calculated from (VIa).

The surface intensities I_p of the image and I_s of the star, in the sensitometer intensity

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* The two spectra compared on the film are: 1, the spectrum of a star of high temperature, modified by passage through the earth's atmosphere, one reflection by a silvered surface and passage through the spectrograph, and 2, the spectrum of the Sun, modified by scattering by and passage through the earth's atmosphere, scattering by the opal glass, reflection by three silvered surfaces and passage through the spectrograph.
Let I_n refer to λ 4701 and I_s to λ 5007 (N_1). Due to the different spectral distribution of a black body of 37000° (star) and one of 6000° (Sun), one would have $I_n/I_s=0.80$. Taking into account the change in spectral distribution of the sunlight when scattered by the sky (Abbot and Fowle, Ann. Astrophys. Obs. Smiths. Inst. II, 155, 1908. Table 36), this is to be multiplied by about 1.30, hence $I_n/I_s=1.04$, so that the previous error is cancelled to a great extent. On the other hand the scattering by the opal glass makes the light redder and the ratio correspondingly smaller again. The additional two reflections of the sky spectrum introduce no appreciable error (Abbot and Fowle, p. 52). On account of the scattering by the opal glass and possible haze in the sky, it seems probable that I_n at 4701 is higher than I_s at 5007, but presumably by not more than 25 per cent of I_s.
** n decreases with ν.

scale, are obtained from measurements of density and the characteristic curve in the manner just described, whereas the area σ of the image and the thickness δ of the star spectrum are measured under the microscope. Thus for any nebular image the observational quantity A_ν is obtained as given by (VI).

As was remarked in section 3.2, the method of spectrophotometry employed enables us to determine only intensity ratios for the same wavelength. Thus in expression (VI) the unit in which I_p and I_s for a given λ are expressed is not known, but the ratio of these quantities, and therefore A_ν, is known.

When the film is fogged by moonlight, the surface intensity I_f of the background is determined at the adjacent portion of the film, and this quantity subtracted from the intensities I_p and I_s, thus correcting for fog. In fig. 2, the fog readings D_f are marked by crosses (+). A smooth curve is drawn through them which gives the fog correction for any desired scale reading n. In fig. 2 it is seen as the lower chain line.

3.4. *Example of measurements and determinations of A_ν: Film No 9, N.G.C. 6543.*—The film No. 9, N.G.C. 6543, will now be discussed in full as an example of the determination of A_ν. The determinations were carried out with the Hartmann wedge microphotometer of the Dominion Astrophysical Observatory. At the centre of the field under the microscope a circular portion of the film of diameter 0·08 mm. is seen magnified; the rest of the field is occupied by the wedge. The two portions of the field are matched, and the average of the two wedge readings is taken as the measure of the density D. The curves needed have already been given in figs. 2, 3a, and 3b. In addition to the measurements which these curves represent, the density D_p of each monochromatic picture was determined in two or more places for each image. The width of the continuous spectrum was also measured under the microscope at a point between H_γ and H_δ. After a slight reduction to be discussed later on, it was δ=0·113 mm., as indicated at the head of Table 4.

Table 4 contains the observational material of film No. 9, and the values of A_ν derived from it. The columns are numbered from 1 to 14. Column 1 contains the designation of the line, 2 its wavelength, 3 the scale reading n from (I), 4 the density D_f of the fog taken from fig. 2, 5 the corresponding intensity I_f from the characteristic curves, figs. 3a and 3b. In column 6 the density D_p of the monochromatic image, measured in two or more places, is given, reduced to the intensities I_p in column 7 by means of the characteristic curves figs. 3a and 3b. Column 8 contains the central density D_s of the star spectrum, and 9 the central intensity I_s derived from it. The fog intensity I_f has to be subtracted from I_p and I_s, and the values of the average I_p and I_s thus corrected are given in columns 10 and 11. The measured area σ of the monochromatic image and the factor $\frac{1}{\lambda}\frac{\delta\lambda}{\delta n}$ from (VIa) are given in columns 12 and 13. The observational quantities A_ν now follow from (VI) by substitution of the corrected I_p (10), I_s (11), σ (12), δ=0·113, and the factor of column 13. They represent the results of the observations to be used in the theory, and are given in column 14.

TABLE 4—EXAMPLE OF MEASUREMENTS AND DETERMINATIONS OF A_ν

FILM No. 9, N.G.C. 6543

Exposure 1h 15m.   Slit width sensitometer spectra 0·30 mm.
δ = 0·113 mm.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Line	λ	n	D_f	I_f	D_p	I_p	D_s	I_s	I_p corr.	I_s corr.	σ	$\frac{1}{\lambda}\frac{\delta\lambda}{\delta n}$	A_ν
Nu......	3726	27·29	28·0	11·2	35·6 / 35·2	66·8 / 62·3	43·3	148·	53·3	137·	0·62	0·01619	0·035
Nu......	3869	29·49	28·1	11·2	42·4 / 41·5	83·2 / 75·9	46·0	110·	68·3	99·	0·62	0·01900	0·068
Hδ......	4102	32·49	29·2	8·2	39·4 / 39·5	24·6 / 24·8	46·9	46·8	16·5	38·6	0·56	0·02103	0·045
Hγ......	4340	35·00	30·7	6·7	44·6 / 45·6 / 45·6 / 46·4	21·4 / 23·4 / 23·4 / 24·8	45·8	23·7	16·5	17·0	0·56	0·02423	0·116
	4701*	38·00	33·2	8·3			48·2	31·6		23·3			
Hβ......	4861	39·11 *D* at 38·77	32·6	11·9	46·1 / 48·2 / 47·6 / 45·7	38·0 / 46·8 / 44·1 / 37·6			29·7	22·9	0·56	0·03145	0·202
N₁......	5007	40·01 *D* at 40·59	26·5	8	53·2 / 53·3	209· / 214·			203·	23·3	0·53	0·03352	1·37
N₂......	4959	39·72							114·	23·3	0·53	0·03284	0·75

Remarks.—Date 1927, Sept. 13-14. Exposure started 0h 35m Pacific Standard Time. At beginning of exposure very thin clouds drifted over, not obscuring nebula. Later clear. The fog on the film is due to moonlight.

Up to H_β the procedure is quite straightforward and needs no further comment. As was remarked before, in Section 3·3, the intensity I_s at the centre of H_β, N_2, and N_1 cannot be determined by interpolation, but instead of this its value is taken for the point λ 4701, marked by a star, which is the nearest point for which it can be well determined.

The images of H_β, N_2, and N_1 overlap. The density of the extreme images H_β and N_1 can well be measured in the region where they are free from N_2, that is in the case considered at n=38·77 for H_β and at n=40·59 for N_1, as indicated in column 3, and the density D_p thus measured is given in column 6 and used for the determination of A_ν.

The only image which gives difficulties is that of N_2, and the way in which the quantity I_p (corr.) in column 10 is obtained needs separate discussion. The density determined at n=39·72 is that of N_2+N_1+fog (see later). One may now assume that the density produced by N_1+fog is the same as that already determined at n=40·59,

PLATE VIII

Fig. 4.—Arrangement of the apparatus for putting on the sensitometer spectra.   Compare **fig.** 1.

Fig. 5.—Mounting of the plane mirror and the concave mirror reflecting the light of the sensitometer on the slit.   Compare fig. 1.

Fig. 6.—Construction of the sensitometer.   The photograph contains also the polished door used for guiding and the film holder.

PLATE VIII

that is $D=53\cdot2$, $53\cdot3$. This density, if produced by N_2 would mean an intensity of N_2: $I=148$, taken from the characteristic curve for N_2, $\lambda4959$, in fig. 3b. In other words, as far as its power of blackening goes, one replaces N_1+fog by a certain, purely fictitious, intensity of N_2, $I=148$. The total blackening $D=55\cdot9$, $57\cdot1$ would correspond to an intensity of N_2, $I=262$ from the characteristic curve of N_2. Subtracting from this total intensity 262 the part which is due to the N_2-equivalent of N_1+fog, one obtains for the actual intensity of N_2 the difference I_p (corr.)$=114$ as given in column 10. The procedure condensed is

$$
\begin{array}{lcccclcc}
 & D & I & & & & D & I \\
N_2 + N_1 + \text{fog} & 55\cdot9 & 229 & 262 & & N_1 + \text{fog} & 53\cdot2 & 148\} \\
 & 57\cdot1 & 295 & & & & 53\cdot3 & 148\} \\
\end{array}\ 148
$$

$$N_1 : I_p = 262 - 148 = 114,$$

the intensities being all taken from the characteristic curve of N_2. Though the procedure is by no means perfect, it appeared the best course under the given circumstances. It will be noted from fig. 3b that the characteristic curves for N_1 and N_2 had to be extrapolated, so that the A_ν for these lines is not as accurate as for the other lines.

During the intermittent guiding as outlined in Section 2, it was noticed in this and all other cases that the main drift was in declination, whereas the adjustment to be applied in right ascension in bringing the nebula back on the centre of the cross was quite small. The slit of the spectroscope was nearly east-west, so that the drift is mainly perpendicular to the slit. This was rather fortunate, since thus the continuous star spectrum remained quite sharp. Partly due to this drift, the monochromatic images are slightly elongated, and have been treated as ellipses. The dimensions of the first two nebular images (though they may not be due to the same source), $1\cdot41 \times 0\cdot56$ mm., have been taken from that of $\lambda3869$ which was best measurable, and the dimensions of the H images, $1\cdot25 \times 0\cdot57$ mm., from $H\gamma$. The major axis of N_2+N_1+fog was measured $1\cdot43$ mm., and subtracting from this the separation of $0\cdot29$ gives $1\cdot14$ mm. for the major axis of each image N_1 or N_2, whereas the minor axis was $0\cdot59$ mm. According to these dimensions, the centre of the image N_2 at $n=39\cdot72$ (Table 3) is completely covered up by N_1, and the density measured there is that of $N_2+ N_1 +$ fog, as stated before, whereas the point $n=40\cdot59$, where N_1 was measured is well outside the N_2 image and the density measured there is that of N_1+ fog and is treated as such in Table 4.

It should further be remarked that the nebulium image, $\lambda3869$, has that of $H\zeta$, $\lambda3889$, superimposed. Since, however, Wright's intensities I_q[6] indicate that the former is predominant, it has been treated as consisting of $\lambda3869$ only.

The width of the star spectrum was determined at $n=38\cdot00$, $\lambda4701$. It appeared to consist of a denser portion of width $0\cdot078$ mm. and intensity $23\cdot3$ and a less dense portion of width $0\cdot064$ mm. and intensity $12\cdot6$, both intensities corrected for fog. The reduced width δ by which the intensity I_s of the densest portion will have to be multiplied to yield the intensity per mm. length is therefore given by $23\cdot3\delta = 23\cdot3\times0\cdot078+12\cdot6 \times0\cdot064$, hence $\delta = 0\cdot113$ mm. Since everywhere else in the continuous spectrum the intensity was determined in its densest portion, the value $\delta = 0\cdot113$ mm. has to figure

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[6] See ⁶.

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as its measured width, as stated before. For the other films, to be discussed in Section 3·5, the value δ is simply the measured width of the star spectrum, without any reduction.

A partial check of some of the wedge photometer measurements seemed desirable, and was carried out with the thermocouple microphotometer at the Mount Wilson Observatory, which was kindly placed at my disposal. The results will be mentioned in the summary of Section 3·5, but will not be made use of, since the material is rather incomplete.

3. 5. *Results of the measurements; Values of A_ν.*—In Table 5 the values of A_ν, determined with the wedge photometer in a manner similar to that fully described for film No. 9 (Section 3·4) are collected.

TABLE 5.—VALUES OF A_ν DETERMINED

Line	λ	N.G.C. 6543				N.G.C. 6572
		No. 7	No. 9 fog	No. 12	Ave.	No. 10
Nu, ($H\zeta$)	3726		0·035		0·035	0·032
Nu_4, ($H\zeta$)	3869	0·054	0·068	0·049	0·057	0·096
Nu_4, H_e	3967	0·028	0·045	0·039	0·033	0·059
$H\delta$	4102	0·034	0·042	0·042	0·040	0·039
$H\gamma$	4340	0·064	0·116	0·105	0·095	0·068
$H e_1$	4471					0·009
$H\beta$	4861	0·101	0·202	0·165	0·16	0·19
N_2 (O III)	4959	0·51	0·75	0·28	0·51	0·84
N_1 (O III)	5007	1·08	1·37	0·93	1·13	0·97
δ (mm.)		0·160	0·113	0·186		0·15
Exp.		1ʰ 15ᵐ	1ʰ 15ᵐ	1ʰ 15ᵐ		0ʰ 45ᵐ
Slit		0·27	0·3	0·3		0·3

The first column gives the designation of the line, the second its wavelength. The origin of the first three nebulium lines, indicated by Nu is as yet unknown. The images of the Nu line 3869 and that of $H\zeta$ 3889 overlap, but Wright's data indicate that the former is the stronger, and it has been treated as Nu 3869. Similarly the A_ν for $\lambda3967$ refers to the total intensity of Nu 3967 and H_ϵ 3970; in this case the two lines are comparable in intensity. The third, fourth, and fifth columns contain the values of A_ν for films Nos. 7, 9, and 12, N.G.C. 6543, and the sixth column their average. The seventh column contains the A_ν values for film No. 10, N.G.C. 6572. Only No. 9 was fogged, as indicated in the table, and the correction for this was determined in the manner described (Section 3·4). In all other cases fog determinations were also carried out for each image, but they showed that fog was absent, so that no correction was applied.

The data for N.G.C. 6543 are obtained independently from three films. The values of A_ν for N.G.C. 6572 were obtained from one film only, No. 10. The A_ν values for He_1 and $H\beta$ were also checked by thermocouple measurements, which gave $0\cdot011$ and $0\cdot22$ for these lines, in good agreement with the wedge measurements.

The photograph Plate IX, fig. 7d, shows that the image of H_β is clearly separated from the compound image of N_1 and N_2. Since the separation of N_1 and N_2 is 0.29 mm., two points about 0.15 mm. from the edge of the compound image on both sides may be considered to consist of N_1 or N_2 only, and thus the density of these two images was measured separately. The result is not very satisfactory, since the values of A_ν are 0.97 for N_1 and 0.84 for N_2, whereas, according to Plaskett's relative intensities[7] the image of N_2 should be fainter than N_1 by a factor of about 2.7. Unfortunately the thermocouple measurements, through an oversight, were carried out in places where presumably the two images overlap. If these measurements are treated as $N_1 + N_2$, the A_ν for thermocouple measurements becomes about 1.7, whereas the total A_ν for the wedge measurements is 1.8*. This total value figures in the theory and seems about right, so that it is decided to use the values of Table 5 as they stand, though the individual values for N_1 and N_2 may be a good deal off.

Two films of N.G.C. 7009, Nos. 6 and 8, were also obtained. However, they are so heavily fogged by moonlight that their result cannot be trusted. The fog correction for the two films is in most cases larger than the surface intensity of the star spectrum corrected for fog. Under those circumstances a fog correction cannot be applied with safety, since the fog at one point is not due to monochromatic light, but to a range of wavelength corresponding to the opening of the collimator formed by the removal of the slit, and hence for a very large correction the results may become illusory. This is probably the explanation why the results of the two films fail to agree. It is thought best not to give the actual values determined, but only mention briefly in Section 5 the provisional conclusions which may be drawn from them, as far as seems permissible.

3.6. *Discussion of errors.*—It is a rather fortunate circumstance that, for the purpose of checking a theory of luminosity and for carrying out an approximate temperature determination based on such a theory, a high accuracy of the quantities A_ν is not required. In fact, the knowledge of the order of magnitude of these quantities seems sufficient for an approximate check.

The accidental errors inherent in the method may best be judged from the material of N.G.C. 6543 in Table 5, which is obtained from three films treated entirely independently. One may conclude from the values of the independent determinations, that the accidental error in the average A_ν for N.G.C. 6543 is about 30 per cent. For N.G.C. 6572 the accidental error may be larger than this, since the values are derived from one film, certainly the individual values of N_1 and N_2, as remarked, are very inaccurate.

An error in part systematic is due to the use of I_s at a shorter wavelength for H_β, N_2 and N_1. As explained in the footnote of Section 3.4, this may lead to a too low value of A_ν for these images, but presumably the error is not more than 25 per cent.

There are, however, serious systematic errors in the present method which should not be underestimated. The intensity per mm. of the star spectrum has been taken as $I_s\delta$, where I_s is the central intensity and δ the measured width; rigorously it should be $\int I_s d\delta$. Now measurements of film No. 10, N.G.C. 6572, with the Mount Wilson ther-

[7] See 37.
* The two values of A_ν may be added, since λ is about the same.

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Fig. 7a.—Film No. 7, N.G.C. 6543. Exp. 1h 13m.

Fig. 7b.—Film No. 9, N.G.C. 6543. Exp. 1h 15m.

Fig. 7c.—Film No. 12, N.G.C 6543. Exp. 1h 15m.

Fig. 7d.—Film No. 10. N.G.C. 6572. Exp. 0h 45m.

Fig. 7e.—Film No. 6, N.G.C. 7009. Exp. 2h 30m.

Fig. 7f.—Film No. 8, N.G.C. 7009. Exp. 2h 0m.

PLATE IX

mocouple showed that the latter quantity, which should be used, is 1·8 times the former. An error of a similar nature is introduced for the total intensity of the monochromatic images which is taken as $I_p\sigma$, σ being the measured area, whereas it should be $\int I_p d\sigma$. Presumably the latter value, which is correct, is also larger than the former.

In the expression (VI) for A_ν, $I_p\sigma$ occurs in the numerator and $I_s\delta$ in the denominator, so that the errors may in part cancel. One might correct the error in the star spectrum by multiplication by a factor of about 1·8, but this has no meaning if a corresponding correction is not applied to the nebular images. It is therefore thought best not to attempt any correction, but to leave the values of A_ν as they stand. However, not too much stress should be laid on the actual values of A_ν, and the possibility should be considered that all the A_ν's given are too high or too low by a factor, which is about the same for all A_ν's and all films, and presumably not exceeding 1·5.*

SECTION 4.—THEORY OF NEBULAR LUMINOSITY. FORMULAE FOR TEMPERATURE DETERMINATIONS

4.1. *The two theoretical mechanisms assumed.*—Mainly through the fundamental work of Hubble contained in two papers[8], it is now generally recognized that a galactic nebula with an emission spectrum is excited to luminosity by radiations of some kind emerging from neighbouring stars of early spectral type. This was first suggested by Russell[9] on theoretical grounds.

The mechanism by which the luminosity in the nebula is excited by stellar radiations may be further specified. Replacing the star by a black body of temperature T, one may assume that atoms in the nebula are first ionized by the absorption of ultraviolet starlight, and subsequently recombine, thus producing a number of line spectra and continuous spectra.

Both Menzel[10] and the author[11] independently conceived of this mechanism of ionization and recombination and calculated the order of magnitude of the brightness of the nebula derived from it, as far as it is due to the Balmer series of hydrogen. Using Hubble's luminosity data for diffuse nebulae, the author derived the approximate temperatures of stars of types B_1, B_0, and O which the mechanism required. The temperatures thus obtained for the three spectral types agreed approximately with the values given by the theory of thermal ionization, and this furnished a check of the theoretical mechanism.

After Bowen[12] had discovered the origin of a number of the so-called nebulium lines, it soon became evident that the most prominent of these lines could not be excited to a sufficient intensity by ionization and recombination, thus necessitating an extension of the theory. The difficulty, and the solution proposed by Bowen in his paper on planetary nebulae[13] may be illustrated by the two prominent lines in any nebular spectrum

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* In this work at the Lick Observatory, Mr. L. Berman, according to a private communication, has determined integrated intensities of nebular images by means of a registering microphotometer. It appears desirable to remeasure the films discussed here in this way and thus possibly eliminate the two sources of error mentioned.

8 E. P. Hubble. Astrophysical Journal 56, 162, 400 (1922).
9 H. N. Russell. Observatory 44, 72 (1921); Proceedings of the National Academy of Sciences 8, 115 (1922).
10 D. H. Menzel. Publications of the Astronomical Society of the Pacific 38, 295 (1926).
11 H. Zanstra. Physical Review 27, 644 (1926, Abstract); Astrophysical Journal 65, 50 (1927).
12 I. S. Bowen. Nature 120, 437 (1927); Publications of the Astron. Soc. of the Pac. 39, 295 (1927).
13 I. S. Bowen. Astrophys. Journal 67, 1 (1928).

of wavelengths 5007 and 4959 Å, usually referred to as N_1 and N_2. According to Bowen's interpretation these lines are due to transitions between two metastable states and the fundamental state in doubly ionized oxygen O^{++} or O_{III}.* The ionization potential of O_{III} is 54·8 volts, or about four times that of hydrogen, whereas the two lines N_1 and N_2 of O_{III} in diffuse nebulae have an intensity comparable with that of the hydrogen line H_β. Where the observed intensity of the hydrogen lines on the basis of the mechanism of recombination requires a temperature of roughly 30,000° for stars associated with diffuse nebulae of the emission type, it is immediately evident that the amount of ultraviolet starlight of that temperature beyond the ionization frequency of O_{III} is entirely inadequate to produce the required intensity of the O_{III} lines by the same mechanism, and that to account on this basis for the intensity of the O_{III} lines would require an absurdly high temperature. Fortunately, however, though the ionization potential of O_{III} is very high, the excitation potential of the O_{III} lines N_1 and N_2, once the O_{III} is present, is very low, namely 2·5 volts. Now the nebula contains abundant free electrons freed by the mechanism of ionization, let us say, from hydrogen, which have an energy sufficient for the excitation of the lines N_1 and N_2 by electron collision. All that is necessary for a line to be strongly excited in this manner is that its excitation potential should be low. As it happens, all such lines of low excitation in nebulae of which the origin is known originate from metastable states.

On this extended hypothesis we have therefore two mechanisms for the excitation of lines in nebulae:

1. *The primary mechanism of ionization and recombination.*—The star is considered as a black body of temperature T. The ultraviolet starlight absorbed beyond the ionizing frequency of a certain atom or ion (e.g. H, He_1, He_{II}) in the nebula causes ejection of photo-electrons, which, on recombination, produce bright line spectra and continuous emission spectra of the atoms concerned.

2. *The secondary mechanism of electron excitation,* due to Bowen. The photo-electrons freed under 1 may give up a great part of their energy in exciting lines of low excitation, before they recombine with the atom in question. This mechanism is presumably applicable to the resonance lines of lowest excitation in the nebular spectrum: the O_{III} lines λ 4959, 5007 (N_2, N_1), the O_{II} lines λ 3726, 3729, the N_{II} lines λ 6548, 6584, and the line of S_{II} recently identified by Bowen[14], λ 6730 which, on his interpretation, should be accompanied by a fainter line at λ 6717.

Thus the luminosity of a number of the so-called nebulium lines becomes a by-product of the ionization and recombination luminosity of lines like those of hydrogen and helium.

Certain spectroscopic data, of interest in the treatment which is to follow, are given in Table 6. The table contains the ionization potentials and corresponding wave-lengths for the atoms and ions that play an important part in nebulae. They are taken from Bowen's paper on planetary nebulae. Carbon is not very important, whereas lines of O_I and N_I have not been observed, which Bowen explains by the very plausible as-

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* In the usual notation, a normal atom is indicated by the suffix I, a singly ionized atom by the suffix II, and so on.

14 I. S. Bowen. Nature 123, 450 (1929).

sumption that the ultraviolet radiation necessary to excite them has all been swallowed up by the abundant hydrogen which has approximately the same ionization potential.

4.2. *Formula for temperature determination from ionization and recombination (H, He₁, He₁₁).* In section 3 the observational quantity A_ν was obtained for each of the monochromatic images measured, defined by the expression

$$A_\nu = \frac{L_p}{\nu} \frac{\delta L_s}{\delta \nu} \qquad (1).$$

L_p represents the total energy in a monochromatic image of frequency ν, and $\frac{\delta L_s}{\delta \nu}$ the total energy per frequency unit of the star spectrum for the same frequency. The quantities A_ν collected in Table 5 provide all the observational material which will now be theoretically elaborated.

The mechanism 1, that of ionization and recombination, will first be discussed. The argument is entirely the same as in the author's paper on diffuse nebulae[15], but is to be adapted to the present use.

The exciting star, that is the central star of the planetary nebula, will be considered as a spherical black body of radius R and surface temperature T. The total energy radiated per second by the star within the frequency interval dv is then *

$$\frac{\delta L_s}{\delta \nu} dv = \pi R^2 c \rho_\nu dv \qquad (2a),$$

where c represents the velocity of light, and $\rho_\nu dv$ the energy density of black body radiation of temperature T, given by Planck's formula

$$\rho_\nu = \frac{8\pi h}{c^4} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1} \qquad (2b),$$

$h = 6.55 \times 10^{27}$, $k = 1.371 \times 10^{16}$, $c = 2.998 \times 10^{10}$.[16]
The total number of quanta emitted per second within the frequency interval dv is obtained by dividing (2a) by $h\nu$, and therefore becomes

$$\frac{8\pi^2 R^2 k^3}{c^2 h^3} T^3 \frac{x^2}{e^x - 1} dx \qquad (3), \text{ where } x = \frac{h\nu}{kT}.$$

The nebular envelope is assumed to surround the star completely, and is considered to consist of atoms of one kind only, to fix the ideas, of atomic hydrogen, which may be ionized. That this simplification "does not materially influence the result, subsequent discussion in Section 5.3 will bear out.

Because of the low density of the incident radiation, one may reasonably expect practically all the hydrogen atoms to be in their normal state. The atomic hydrogen in its normal state is capable of absorbing all the energy of a wavelength shorter than that of the head of the Lyman series, frequency $\nu_0 = 32.84 \times 10^{14}$ per second. Assuming

* (2a) and (2b) are (1a) and (1b) in the former paper.
[15] See [1].
[16] R. T. Birge. Probable Values of the General Physical Constants. Physical Review Supplement, Vol 1, No. 1 (1929).

a sufficient number of normal atoms in the envelope, so that the absorption is complete, the number of quanta absorbed per second from the ultraviolet starlight is

$$N_{ul} = \frac{8\pi^2 R^2 k^3}{c^2 h^3} T^3 \int_{x_0}^\infty \frac{x^2}{e^x - 1} dx \qquad (4), \text{ where } x_0 = \frac{h\nu_0}{kT} \qquad (4a),$$

according to (3).

Assuming a stationary state in the nebula, this absorption of ultraviolet light is accompanied by N_{ul} ionizations and recombinations per second. It is the recombination on the different levels of the atom that results in the re-emission of a number of line spectra and continuous spectra. Let the number of quanta thus emitted per second in the Balmer series be Ba, in the continuous spectrum at its head Ba_c, and in the continuous spectrum at the head of the Lyman series Ly_c, the latter being not observable. Then the author has shown that

$$Ly_c + Ba_c + Ba = N_{ul} \qquad (5),$$

which expression is based on the reasonable assumption that the absorption coefficient of the first few lines of the Lyman series is large compared with the absorption coefficient of the continuous spectrum originally absorbed. In Section 7.1 it will be shown that this assumption is theoretically justified.

In expression (1) the quantity L_p represents the total energy emerging from one monochromatic image of frequency ν. The total number of quanta N_p emerging from such an image is L_p divided by $h\nu$, or, according to (1)

$$N_p = \frac{L_p}{h\nu} = A_\nu \frac{1}{h} \frac{\delta L_s}{\delta \nu} \qquad (6).$$

Substituting (2a) and (2b) in (6) and changing to the variable x, one obtains

$$N_p = \frac{8\pi^2 R^2 k^3}{c^2 h^3} T^3 \frac{x^3}{e^x - 1} A_\nu \qquad (7), \text{ where } x = \frac{h\nu}{kT}.$$

If now we sum over all lines in the Balmer series, we have $\Sigma N_p = Ba$, to be substituted in (5). To include the line H_α not measured, the observations of Plaskett for the ratio H_α / H_β will be used. Strictly speaking we should also have a measure for the quantities Ba_c and Ly_c, but in the present work this was not possible, and they have therefore been neglected, which procedure, as regards order of magnitude, seems justified. The equation (5) becomes therefore $\Sigma N_p = N_{ul}$, and this, with (7) and (4), gives

$$N_{ul} = \Sigma N_p$$

or

$$\int_{x_0}^\infty \frac{x^2}{e^x - 1} dx = \Sigma \frac{x^3}{e^x - 1} A_\nu \qquad (8), \text{ where } x = \frac{h\nu}{kT}$$

where $Ba_c + Ly_c$ has been neglected with respect to Ba.

The approximate equation (8) is the formula for temperature determination from ionization and recombination, according to the present method. It is solved by trial. The quantities A_ν are known from observation (Table 5). For any value of T tried

the left-hand side and the right-hand side assume therefore definite values, and that value of T for which both sides are equal represents the determined temperature of the star. The actual determinations are carried out in Section 5. The second column of Table 7 gives the value of the integral needed in (8) as a function of x_0, obtained by Debye's method (Section 7.2).

4.3. *Formula for temperature determination from electron excitation ("Nebulium").*—From the observed sizes of the monochromatic images there seems no doubt that the most prominent "nebulium" lines, assumed to be due to electron impact, are all excited in or near the region where the recombination of hydrogen takes place. For calculating the intensity of the nebulium resonance lines, as produced by mechanism 2 of electron impact, the following simplification is therefore introduced. The star is again conceived of as a black body of temperature T and the nebular envelope is considered to consist of normal and ionized hydrogen with some ions, chiefly O_{II}, O_{III}, and N_{II}, intermixed. Disregarding the absorption by other atoms or ions, it is further assumed that the normal hydrogen absorbs the ultraviolet starlight from the frequency ν_0 of the head of the Lyman series, up to infinity. In how far this simplification influences the result will be examined in Section 5·3.

The energy E_{ul} absorbed per second by the normal hydrogen from the ultraviolet starlight is, according to (2a) and (2b)

$$E_{ul} = \int_{\nu_0}^{\infty}\frac{\delta L_s}{\delta\nu}\,d\nu = \frac{8\pi^2 R^2 k^4}{c^2 h^3}T^4\int_{x_0}^{\infty}\frac{x^3}{e^x-1}\,dx \qquad (9), \text{ where } x = \frac{h\nu}{kT},$$

assuming, as before, the absorption to be complete.

Part of this energy, E_i, is required for ionization, the rest E_f is the energy of the free electrons when liberated. Since there are N_{ul} photo-electrons produced per second, each requiring one quantum $h\nu_0$ for ionization, we have $E_i = h\nu_0 N_{ul}$, and

$$E_f = E_{ul} - h\nu_0 N_{ul} \qquad (10).$$

Substituting (9) for E_{ul} and (4) for N_{ul}, we have, for the energy of the photo-electrons freed per second

$$E_f = \frac{8\pi^2 R^2 k^4}{c^2 h^3}T^4\left\{\int_{x_0}^{\infty}\frac{x^3}{e^x-1}\,dx - x_0\int_{x_0}^{\infty}\frac{x^2}{e^x-1}\,dx\right\} \qquad (11).$$

This energy is available for the excitation of the resonance nebulium lines by the mechanism 2 of electron impact.

Equation (7) gives the number of quanta N_p emerging per second from one nebulium image, frequency ν. The energy emitted per second by such an image is therefore $h\nu N_p$, and the total energy E_e emitted per second by the nebula in the nebulium lines is $\Sigma h\nu N_p$, or, changing to the variable x,

$$E_e = \frac{8\pi^2 R^2 k^4}{c^2 h^3}T^4\sum\frac{x^4}{e^x-1}A_\nu \qquad (12),$$

expressed in terms of the observational quantity A_ν. If now one assumes that the electrons, before they recombine with the hydrogen ions, will have lost most of their kinetic energy in exciting the nebulium lines, one has approximately $E_f = E_e$.

TABLE 6.—IONIZATION POTENTIAL V_0 IN VOLTS AND CORRESPONDING WAVELENGTH λ_0 IN Å FOR ATOMS AND IONS IN NEBULAE

	H	He$_1$	N$_{II}$	O$_{II}$	N$_{III}$	He$_{II}$	O$_{III}$	O$_{IV}$
V_0	13·54	24·41	29·56	35·00	47·2	54·18	54·8	77·0
λ_0	912	506	418	353	262	228	225	160

Remark.—The table contains only the data of interest in the present treatment. We have

$$x_0 = \frac{h\nu_0}{kT} = 1.432 \times 10^8/\lambda_0 T = 11600\ V_0/T.$$

The quantity x_0 is used for temperature determination according to Sections 4·2 and 4·3.

The equation for temperature determination by the mechanism of electron impact becomes therefore, on account of (11) and (12),

$$E_f = E_e,$$

or

$$\int_{x_0}^{\infty}\frac{x^3}{e^x-1}\,dx - x_0\int_{x_0}^{\infty}\frac{x^2}{e^x-1}\,dx = \sum\frac{x^4}{e^x-1}A_\nu \qquad (13),$$

The summation is to be extended over the nebulium resonance lines, mentioned under mechanism 2 (Section 2).

Table 7, columns 3 and 2, gives the values of the two integrals I_2 and I_1, occurring on the left-hand side, as functions of x_0. They were obtained by Debye's method (Section 7.2). Column 4 of the same table contains the left-hand side $I_2 - x_0\ I_1$ of (13) as a function of x_0.

The equation (13) is solved by trial. The values of A_ν are known from observation (Table 5). For different trial values of T the left-hand side and the right-hand side of (13) can therefore be evaluated, and the value of T for which both sides are equal represents the determined temperature of the star.

Due to the simplification that the hydrogen has been assumed to absorb all the ultraviolet starlight, and the fact that not all resonance lines could be considered, the resulting temperatures are only approximate. They represent minimum values for the temperature in any case.

The actual temperature determinations are carried out in Section 5.

TABLE 7.—INTEGRALS USED FOR TEMPERATURE DETERMINATOINS

x_o	I_1	I_2	$I_2 - x_o I_1$	$T/1000°$ (x_o for H_f)
0·0	2·4041	6·494	6·494	
0·2	2·3854	6·491	6·014	785·0
0·3	2·3634	6·486	5·777	523·0
0·6	2·2574	6·437	5·083	262·0
1·0	2·0502	6·269	4·219	157·0
1·5	1·7387	5·878	3·270	105·0
2·0	1·4180	5·318	2·482	78·5
2·5	1·1204	4·649	1·848	62·8
3·0	0·8623	3·942	1·355	52·3
3·5	0·6492	3·251	0·979	44·9
4·0	0·4797	2·617	0·698	39·25
4·5	0·3487	2·062	0·493	34·9
5·0	0·2500	1·594	0·344	31·4
6·0	0·1241	0·9081	0·163	26·2
7·0	0·05930	0·4908	0·0757	22·4
8·0	0·02751	0·2543	0·0342	19·6
9·0	0·01246	0·1274	0·0155	17·4
10·0	0·00554	0·0620	0·0066	15·7
11·0	0·00242	0·0295	0·0029	14·3

$$I_1 = \int_{x_o}^{\infty} \frac{x^2}{e^x - 1} dx. \qquad I_2 = \int_{x_o}^{\infty} \frac{x^3}{e^x - 1} dx.$$

SECTION 5.—DETERMINATION OF STELLAR TEMPERATURES BY THE FOREGOING METHODS. COMPARISON OF THEORY AND OBSERVATION.

5.1. *Results of the temperature determinations by the foregoing methods.*—The formulae for temperature determination (8) and (13) obtained in Section 4 will now be applied to the observational material of Table 5.

The variable x contained in these formulae is defined by (3a). Introducing $\lambda = \frac{c}{\nu}$, and substituting the numerical values of the universal constants, this becomes

$$x = \frac{h\nu}{kT} = 1·432 \times 10^8/\lambda T \qquad (14),$$

where λ is expressed in Å and the absolute temperature T in degrees centigrade. The lower limit of integration, by the same expression, becomes

$$x_o = \frac{h\nu_o}{kT} = 1·432 \times 10^8/\lambda_o T = 11600 \, V_o/T \qquad (14a),$$

where V_o is the ionization potential in volts and λ_o the corresponding wavelength as given in Table 6.

Table 8 gives the temperature determinations for the two nebulae by the mechanism 1 of recombination applied to the Balmer series of hydrogen. As explained at the head of the table, the equation (8) to be solved for T is written in the form

$$I_1 - \Sigma_3 = \Delta, \text{ with } \Delta = 0,$$

where I_1 stands for the integral on the left-hand side and Σ_3 for the summation on the right-hand side. The second column contains a set of values x_o, the first column the corresponding T from (14a) in thousands of degrees; and the third column the corresponding values of the integral I_1 obtained directly from Table 7. For a certain value of T in the first column, the x for a hydrogen line, wavelength λ, is obtained from (14), whereas Table 5 gives the observational quantity A, for this line. Thus, for each line,

$$\frac{x^3}{e^x - 1} A_\nu,$$

is obtained, which, summed over all lines of the Balmer series, gives Σ_3. For N.G.C. 6543, column 4 contains Σ_3, and column 5 the difference Δ of I_1 and Σ_3. The temperature determined is the value of T for which Δ = 0 and is found 39,000° by interpolation between the values of Δ in column 5 and the values of T in column 1. The interpolation is almost linear, but was carried out graphically. An idea of the accuracy of the method may be obtained by multiplying all determined values A_ν by 2; this makes all Σ_3 twice as large, and the resulting temperature would be 47,000°. Similarly division of all A_ν's by 2 would lead to a temperature of 33,000°. The temperatures determined for N.G.C. 6543 and N.G.C. 6572 are given at the bottom of the table and also, in parentheses, the values for the two extreme cases mentioned.

TABLE 8.—TEMPERATURE DETERMINATIONS FROM RECOMBINATION: H

$$\lambda_o = 912 \text{ Å}, \quad x_o = 1·570 \times 10^5/T \text{ from (14a)}.$$

Formula: (8) $I_1 - \Sigma_3 = \Delta$, $\Delta = 0$, $\Sigma_3 = \Sigma \frac{x^3}{e^x - 1} A_\nu.$

Lines: $H_\beta, H_\gamma, H_\delta$. Assumed number of quanta in $H_\alpha = 5·4 \times$ that in H_β (Plaskett, Orion Neb.), and number of quanta beyond H_δ = that of H_δ.

I_1 from Table 7, x from (14), A, from Table 5.

$T/1000°$	x_o	I_1	N.G.C. 6543		N.G.C. 6572	
			Σ_3	Δ	Σ_3	Δ
31·4	5·0	0·25	0·66	−0·41	0·74	−0·49
34·9	4·5	0·35	0·56	−0·21	0·64	−0·29
39·25	4·0	0·48	0·47	+0·01	0·53	−0·05
44·9	3·5	0·65	0·38	+0·27	0·43	+0·22
52·3	3·0	0·86	0·29	+0·57	0·33	+0·53
			$T = 39,000°$		$T = 40,000°$	
			(× 2, 47,000°)		(× 2, 49,000°)	
			(÷ 2, 33,000°)		(÷ 2, 34,000°)	

In taking the summation Σ_3 for the lines of the Balmer series, only the A_ν values for H_β, H_γ, and H_δ were available. H. H. Plaskett's[17] determinations of relative intensities give for the ratio of the number of quanta N_p in H_α to that in H_β the value $N_{p\alpha}/N_{p\beta} = 5\cdot4$ for the Orion Nebula (N.G.C. 1976). Assuming this ratio to be about the same for the nebulae under consideration, one obtains the N_p for H_α by multiplying the N_p for H_β by $5\cdot4$*. Now formula (7) shows that the quantity $\frac{x^3}{e^x-1} A_\nu$ may be looked upon as the number of quanta N_p expressed in the unit $8\pi^2 R^2 k^3 T^3_i / c^2\, h^3$, and hence its approximate value for H_α is obtained by multiplying its value for H_β by $5\cdot4$. Thus e.g. for N.G.C. 6543, the value of Σ_3 for $T/1000° = 31\cdot4$ is calculated as follows by (14) and Table 5:

Line	H_δ	H_γ	H_β	H_α
$\frac{x^3}{e^x-1} A_\nu$	0·027	0·059	0·085	0·085 × 5·4 = 0·460

To take the lines higher than H_δ into account, the value 0·027 for H_δ has been added once more to it (though this correction is rather immaterial), and so the total Σ_3 becomes 0·66 as given in the fourth column of Table 8. The figures show also that for the trial temperature $T/1000° = 31\cdot4$ the ratio of the number of quanta in H_β to that in H_γ would be $N_{p\beta}/N_{p\gamma} = 1\cdot4$, and it is about the same for the determined temperature $T/1000° = 39$. Now Plaskett's relative intensities for the Orion Nebula give $2\cdot4$ for this ratio. This no doubt is due to the fact that the image of H_β in the present work has been measured near its edge (due to the overlapping by N_1 and N_2), which results in a too low measured value. The determined temperature 39,000° from H for N.G.C. 6543 is therefore too low. For the nebula N.G.C. 6572 the image of H_β is well separated and here the observations give $N_{p\beta}/N_{p\gamma} = 2\cdot3$, in good agreement with Plaskett's value $2\cdot4$, so that the resulting temperature for H, 40,000° is more reliable.

Table 9 gives the temperature determination for N.G.C. 6572 by the mechanism 1 of recombination applied to the spectrum of $H\zeta$. Of this spectrum only one line $\lambda4471$ could be observed. It was assumed that the number of quanta emerging per second N_{ui} from this line was from 1/5 to 1/20 of the number of recombinations per second N_{ui}, and accordingly that Σ_3 is from 5 to 20 times the contribution of the one line $\lambda4471$. The resulting temperature is between 35,000° and 42,000°, as indicated at the bottom of the table. These values are, of course, intended as a very approximate estimate rather than an actual determination.

TABLE 9.—TEMPERATURE DETERMINATION N.G.C. 6572 FROM RECOMBINATION: $H\zeta$

$\lambda_o = 506 \; Å, \; x_o = 2.83 × 10^5/T$ from (14a).

Formula: (8) $I_1 - \Sigma_3 = \Delta, \; \Delta = 0, \; \Sigma_3 = \Sigma \frac{x^3}{e^x - 1} A_\nu$.

I_1 from Table 7, x from (14), A_ν from Table 5.

Line: $\lambda 4471$. To get N_{ui} from the number of quanta emitted per second in this one line, assume a factor between 5 and 20.

N.G.C. 6572

$T/1000°$	x_o	I_1	$\frac{x^3}{e^x-1} A_\nu$	Δ factor 5	Δ factor 20
31·5	9·	0·012	0·0053	-0·015	-0·094
35·4	8·	0·028	0·0045	+0·005	-0·062
40·5	7·	0·059	0·0037	+0·040	-0·015
47·2	6·	0·124	0·0029	+0·110	+0·066
				$T=35,000°$	$T=42,000°$

Table 10 contains the temperature determinations for the two nebulae by the mechanism 2 of electron excitation. The equation (13) to be solved for T is written in the form

$$I_2 - x_o I_1 - \Sigma_4 = \Delta, \quad \text{with } \Delta = 0.$$

Since the photo-electrons are assumed to be freed from hydrogen (Section 4.3), the x_o is that corresponding to the ionization of hydrogen by (14a). The value $I_2 - x_o I_1$ is obtained from Table 7, and the x for the different lines under the summation sign from (14), where the notation Σ_4 indicates that the x occurs to the power 4. After the explanation given for table 8, the method of solution needs no further comment. As shown by Table 5, the result almost exclusively depends on the O_{III} resonance lines N_1 and N_2. The other lines included are the O_{II} resonance line $\lambda 3726$, the line $\lambda 3869$, and one-half of the intensity of $\lambda 3967$, since in part it is due to $H\zeta$. The first of these three lines (containing also $\lambda 3729$) should certainly be included in the theoretical mechanism, the other two are of unknown origin, but the fact that their intensity is comparable with that of the resonance line $\lambda 3726$ makes it likely that they are excited by the same mechanism. However, as remarked, the influence of these lines is not important.

TABLE 10.—TEMPERATURE DETERMINATION FROM ELECTRON EXCITATION (NEBULIUM)

Photo-electrons from hydrogen: $\lambda_o = 912 Å$, $x_o = 1.570 × 10^5/T$ from (14a).

Formula: (13) $I_2 - x_o I_1 - \Sigma_4 = \Delta, \; \Delta = 0, \; \Sigma_4 = \Sigma \frac{x^4}{e^x - 1} A_\nu$.

$I_2 - x_o I_1$ from Table 7, x from (14), A_ν from Table 5.
Lines: $\lambda 3726, 3869, N_2, N_1$, and $\frac{1}{2}$ of $\lambda 3967$.

The most significant feature of Table 11 is perhaps the good agreement for each nebula between the temperature based on the recombination of hydrogen and that based on the strength of the nebulium lines. On the theory proposed, the number of quanta observed in the Balmer series of hydrogen is a theoretical measure for the number of ultraviolet quanta in the starlight beyond the head of the Lyman series, whereas the energy in the resonance nebulium lines is a theoretical measure for the energy of the photo-electrons freed from hydrogen by the same ultraviolet starlight. When, on this interpretation, both the number of ultraviolet quanta and the energy of the photo-electrons it is capable of producing lead to nearly the same temperature, the conclusion seems hardly avoidable that the interpretation is essentially correct, and that the stellar radiation for a considerable region beyond the head of the Lyman series is approximately that of a black body of the temperature determined, of which the region in the photographically-observed star spectrum forms approximately a part.

TABLE 11.—SUMMARY OF THE TEMPERATURES OBTAINED BY VARIOUS METHODS

Method	N.G.C. 6543	N.G.C. 6572	N.G.C. 7009
H	39,000°*		(55,000°)
He_{I}		40,000°	
He_{II}		35,000° to 42,000°	(70,000°)
Nebulium......	37,000°	38,000°*	(50,000°)

*Less accurate determinations are marked by a star, the very inaccurate determinations for N.G.C. 7009 have been put in parentheses.

It is realized that the assumption that the central star is a black body may be only a rough approximation, since it seems certain that absorptions in the far ultraviolet may occur, and other unknown factors may influence the energy distribution. Also the other assumptions regarding the physical mechanisms involve several approximations, to be considered in 5.3. Nevertheless, it appears from the approximate agreement of the temperatures derived by different methods from the luminosity data, that the assumption that the star behaves as a black body and the luminosity is produced by known physical mechanisms offers a fair interpretation of the facts.

5.3. *More detailed discussion of the mechanisms involved and its influence on the temperatures determined.*—In the derivation of the formulae for the temperature determination in Sections 4·2 and 4·3 a simplified view of the nebula has been taken. The question may now be considered as to how far the simplifications introduced influence the values of the temperatures determined.

In the slitless spectra of planetaries, the monochromatic images due to different types of atoms and ions have different sizes, so that it appears that the stellar radiation passes through different successive shells, partly overlapping, in each of which a certain type of excitation is predominant. Russell called attention to the fact that for some of

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$T, 1000°$	x_0	$I_2 - {}_t I_1$	N.G.C. 6543 ϵ_1	N.G.C. 6543 Δ	N.G.C. 6572 ϵ_1	N.G.C. 6572 Δ
31·4..........	5·0	0·34	0·86	-0·52	0·98	-0·64
34·9..........	4·5	0·49	0·66	-0·17	0·76	-0·27
39·25.........	4·0	0·70	0·49	+0·21	0·56	+0·14
44·9..........	3·5	0·98	0·35	+0·63	0·40	+0·58
52·3..........	3·0	1·35	0·23	+1·12	0·26	+1·09
			$T = 37,000°$		$T = 38,000°$	
			(×2, 42,000°)		(×2, 43,000°)	
			(÷2, 33,000°)		(÷2, 33,000°)	

The two films of N.G.C. 7009, as mentioned in Section 3.5 gave very unsatisfactory results, and the values of A_ν have therefore not been given. Taking the average values for the two films as they were obtained, the temperature from the recombination of hydrogen becomes about 55,000°, and that from the electron excitation of the nebulium lines about 50,000°. An error of 10,000° is quite possible. For the line λ 4686 of He_{II} $(\lambda_0 = 228 \text{ Å})$ it was assumed that the number of quanta is about one-half of the number of recombinations N_{ul}.* The resulting temperature is about 70,000°, where the error may well be 5,000°. Though the temperatures obtained for this nebula are very inaccurate, there is no doubt that the determinations from hydrogen and nebulium give a temperature decidedly lower than that of He_{II}, the difference being roughly 20,000°.

The result of the temperature determinations are collected in Table 11. This table represents the main results of the present work. The H determination for N.G.C. 6543 and the nebulium determination for N.G.C. 6572 are less accurate, since the ratios $H/\beta H_\gamma$ (this section) and the ratio N_1/N_2 (section 3.5) for these cases do not agree with Plaskett's ratio for the Orion Nebula. These less accurate values are marked by a star. The very inaccurate values for N.G.C. 7009 are put in parentheses. For this nebula, a re-determination from photographs which are not fogged is desirable.

5.2 *Comparison of theory and observation.*—For comparison of the theory with observation, the summary in Table 11 may serve. Since there is an approximate agreement between the temperatures derived by the different methods all based on luminosity data, it appears that the mechanism proposed is capable of calculating at any rate the order of magnitude of the effects.

The most serious discrepancy occurs for N.G.C. 7009 between the temperatures derived from He_{II} and the much lower temperatures derived from H and nebulium. As Professor Bowen pointed out to me, the place which this nebula occupies in the sequence of Table 2 in his paper on planetary nebulae[18] based on Wright's intensities I_ν, indicates an abnormally strong intensity of He_{II}, so that this nebula is perhaps an extreme case.

* Taking the case of hydrogen, Plaskett's relative intensities for the Orion Nebula give for the number of quanta in H_α about 0·7 times the number of quanta in the Balmer series, hence 0·7 N_{1t}. Assuming the same ratio for the lines in the Paschen series, the first line in this series would emit (0·7)² N_{ν} or 1·2 N_{ν}) quanta per second. To this line in H, λ 4686 in He_{II} is analogous.
18 See 13.

the prominent images the size of the shells where the excitation takes place increases, as the ionization potential of the substance in the shell decreases. For the succession of shells of the different ions of oxygen, Bowen has offered a theoretical explanation, giving in this also evidence for the assumption due to him that the nebulium resonance lines are excited by electron impact. Actually the shells due to different elements do not quite follow the succession which their ionization potential would indicate, in particular the hydrogen shell is rather small and seems to interpenetrate other shells of higher ionization potentials, which, according to Bowen, may be due to the fact that it can only be singly ionized.

It is hard to form a clear picture of the sizes of the different shells and in this respect a systematic survey of the observed sizes would be desirable. It seems, however, clearly indicated that the He_{II} and O_{III} shell (meaning by this where the recombination of these substances takes place), and perhaps the N_{III} shell, are definitely inside that of hydrogen. The shells of He_{I}, N_{II} (indicated according to Bowen by the N_{III} electron excitation lines), O_{II} (O_{III} electron excitation shell), and O_{I} (O_{II} electron excitation shell), though having different sizes, are probably largely intermixed with the hydrogen.

Mechanism 1. In deriving the formula (8) for the temperature from recombination of hydrogen, it has been assumed that the number of photo-electrons freed is equal to the number of quanta beyond the head of the Lyman series in the ultraviolet starlight, as indicated by the limits x_0, and ∞. However, the ultraviolet radiation, before reaching the hydrogen, has been transformed by the substances O_{III}, He_{II} and N_{III} of the inner shells, and, in part, by the substances O_{II}, N_{II}, and He_{I} more or less intermixed with it; also by O_{I}, but this influence is not appreciable, as Bowen concludes from the absence of O_{I} recombination lines. Energy diagrams of O_{III}, He_{II}, O_{III}, and He_{I}, show that, as the electron falls back on the different levels, it always produces one and no more than one line of wavelength shorter than 912 Å*, the ionizing wavelength of hydrogen. Hence, as Bowen has remarked, for every quantum which they take away from the ultraviolet radiation, these substances return another ultraviolet quantum suitable for ionizing the hydrogen. On the other hand, as follows from the energy diagrams and intensity estimates of lines in the laboratory, the substances N_{III} and N_{II}, make only a partial return, and presumably much of the radiation absorbed by them is lost for the hydrogen. Considering everything, it seems best to leave the limits x_0 and ∞ in (8) as they stand.

In deriving (8) from (5), the quantities $B\alpha_c$ and Ly_c in the latter formula have been neglected. The continuous spectrum at the head of the Balmer series observed in the three planetaries under consideration, shows that the quantity $B\alpha_c$ is not negligible, and presumably still less Ly_c. What fraction of the number of recombinations has thus been neglected is hard to say, but it is certain that this number is larger than assumed, and hence the determined temperature is too low. Even if it is assumed that the value of Σ_3 used should be multiplied by 2 on this account, the temperature for N.G.C. 6543 becomes 47,000° instead of 39,000°, and for N.G.C. 6572, 49,000° instead of 40,000°. It is therefore believed that the values determined, though they are lower limits, may be considered as approximate values of T. Since electrons of low velocity are presumably

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*It is justified to include lines originating from metastable states.

captured more by the higher levels, the approximation is best for lower temperatures, as when the continuous spectrum at the head of the Balmer series is probably faint.

What has been said for the spectrum of hydrogen applies to a certain extent also to the recombination spectra of He_{II} and O_{III}. Also, according to Bowen, an appreciable portion of the ultraviolet light for the excitation of He_{II} is taken away by O_{III} since the ionization potential of these two ions is about the same (Table 6). so that the ultra-violet radiation is divided among these two types of ions. Especially due to this cause the temperature derived from He_{II} without taking account of O_{III} should be somewhat low.

Mechanism 2.—In deriving formula (13) for the temperature from electron excitation of the resonance nebulium lines, the nebular envelope was considered to consist of hydrogen, with some ions, chiefly O_{II}, O_{III} and N_{II} intermixed.

Now, as has been stated, there are actually the shells of He_{II}, O_{III} and presumably N_{III} inside the H shell. Assuming that all the ultraviolet radiation beyond the ionizing frequency of N_{III}, corresponding to 47·2 volts (Table 6) would be lost, the upper limit of integration in (13) would not be ∞, but $x_0 = 47·2\ x_0/13·54 = 3·48\ x_0$. Denoting the old temperatures by T, the new temperatures by T', one has $T = 52·3$ for $T = 51·8$, and $T' = 62·8$ for $T = 57·7$, in thousands of degrees. For higher temperatures one should probably also take the photo-electrons freed from N_{III}, He_{II}, and O_{III} into account for the production of the nebulium lines, and in general add up the photo-electrons due to different frequency bands in a manner as outlined by Bowen. For nebulium temperatures up to 50,000° the influence of the inner shells does not appear to be of importance.

The hydrogen shell itself, as stated before, is presumably largely intermixed with the shells of He_{I}, N_{II}, and O_{II}. Of the ultraviolet radiation, the part from $\lambda = 912$ Å to $\lambda = 506$ Å, the ionizing wavelength of He_{I} (Table 6), is entirely available for the hydrogen. For the shorter wavelengths there will be a competition between H and the other substances, but it is believed that here also the hydrogen, through its greater abundance, may get a large fraction. For temperatures up to 50,000° the presence of the other substances is the first factor which makes Σ_4 as determined too low. The second factor is that not all nebulium resonance lines, in particular those of N_{III}, have been taken into account, and the third factor that not all the energy of the photo-electrons when liberated ultimately is used for exciting the nebulium resonance lines. To take all the different factors into account requires separate investigation, preferably based on more observational data. The situation is extremely involved, but one may estimate that the factor by which Σ_4 as determined in the present investigation has to be multiplied may well be as large as 2.

Summarizing the foregoing considerations, one may say that the temperatures as derived from hydrogen are too low, mainly because $B\alpha_c$ and Ly_c in (5) have been neglected. The temperatures derived from nebulium by electron excitation are also too low, in the first place because other substances than H are present, in the second place because not all nebulium lines have been taken into account, and in the third place because not all the energy of the photo-electrons is given up in excitation. For the simplified method on which (8) and (13) are based, it was noted that the determinations from hydrogen

and nebulium agreed well. Refinements as outlined here will make the temperature by both methods higher, but it is to be expected that the agreement will still be there, especially for temperatures around 40,000°, though it may not be as good.

It is realized that in the present work only a first attack on the problem of luminosity in planetary nebulae and stellar temperatures has been made, and only a treatment as regards order of magnitude has been attempted. The situation in a planetary nebula is much more involved than in the simplified picture on which the temperature determinations are based, and at higher temperatures this no doubt leads to discrepancies, though it is not so serious for the H and nebulium determinations around 40,000°. A clear insight into the actual situation, so that the theoretical assumptions can be more specified, will no doubt require a large amount of observational work of different nature, and it has been thought best not to introduce refinements in the treatment, since that would involve introducing detailed assumptions without sufficient observational evidence.

In Section 6 the nebulium method will be applied to higher temperatures. It may be noted, after the considerations just given, that such temperatures derived on the simplified assumptions for nebulium are generally too low, and that from about 60,000° on it is better to look upon them only as lower limits required by the mechanism discussed. For temperatures of 40,000° and lower the method should be fairly accurate, but of course the higher temperatures as considered in Section 6 are in many ways more interesting.

SECTION 6.—AN APPROXIMATE TEMPERATURE DETERMINATION BASED ON THE DIFFERENCE IN MAGNITUDE OF STAR AND NEBULA

6.1. *Formula for the temperature determination from the difference in magnitude of star and nebula.*—The temperature determination based on the mechanism 2 of electron excitation may be extended so as to include a great number of cases for which data are already available.

According to Brill*, the photographic brightness B_* of a star observed at zenith may be considered proportional to the energy output per frequency unit $\dfrac{\delta L_s}{\delta \nu}$ of a black body of the same radius and temperature at a certain wavelength λ_{ph}, the isophotic wavelength for the photographic brightness.

According to (2a) and (2b) we have therefore

$$B_* = \frac{8\pi^2 R^2 h}{c^2}\, \frac{\nu_{ph}^3}{e^{x_{ph}} - 1}\, C_1 \quad (15), \qquad x_{ph} = \frac{h\nu_{ph}}{kT},$$

where C_1 is a constant independent of T and R.

The visual or photographic brightness B_n of the nebular envelope B_n is in general mainly due to the two strong nebulium lines N_1 and N_2 in the green. This statement only holds well for the visual brightness on which the determinations in section 6.2 are based. Nevertheless for an orientation, the photographic brightness might also serve. These two lines contain also the major part of the energy in the resonance lines due to electron excitation, which has been assumed to be approximately equal to the energy E_f of the free electrons given by (11). According to this formula we have therefore, putting $T = \dfrac{h\,\nu_o}{k\,x_o}$, approximately

* See Section 6.3. Brill's definition of isophotic wavelength refers to the energy output per wavelength unit, and only reduces to the above when λ_{ph} is constant.

$$B_n = \frac{8\pi^2 R^2 h}{c^2}\, \nu_o^4\, \frac{1}{x_o^4}\, (I_2 - x_o I_1)\, C_2 \quad (16),$$

where C_2 is a constant independent of T and R, and I_1 and I_2 the abbreviations for the integrals as given in Table 7.

If m_n is the photographic or the visual brightness of the nebula, and m_* the photographic magnitude of the star, we have by definition

$$m_* - m_n = 2\cdot5 \log \frac{B_n}{B_*}$$

Substituting (15) and (16), and noting that neither the isophotic frequency nor the ionization frequency ν_o of hydrogen depends on T or R, we have

$$m_* - m_n = 2\cdot5 \log \left[\frac{1}{x_o^4}\, (I_2 - x_o I_1)\, (e^{x_{ph}} - 1) \right] + 20 + C \quad (17),$$

where C is a constant independent of the temperature T and the radius R of the star. The additive constant has been put $20 + C$ in order to make the values of $m_* - m_n - C$ occurring in Table 12 positive, merely for the sake of convenience.

For the purpose of the present investigation, the value $\lambda_{ph} = 4210$ Å may be adopted for O stars,* and since λ_o is 912 Å, we have

$$x_{ph} = 0\cdot2166\, x_o, \quad (17a).$$

For different values of x_o, the value of $I_2 - x_o I_1$ is given by Table 7, x_{ph} by (17a), and T by (14a) or the last column of Table 7. Thus $m_* - m_n - C$ is obtained by substitution in (17) for different temperatures. Since the temperature scale in Table 7 is not equidistant, the values of $m_* - m_n - C$ were plotted against T, and from the graph these values determined for more convenient values of T. The result is given in Table 12. This table is the mathematical equivalent of (17) and (17a).

TABLE 12.—VALUES OF $m_* - m_n - C$ FOR DIFFERENT TEMPERATURES

T/1000°	15	20	25	30	35	40	45	50	55	60
$m_* - m_n - C$	6·2	9·1	11·0	12·3	13·3	14·0	14·7	15·2	15·7	16·1
T/1000°	70	80	90	100	120	150	250	500	750	
$m_* - m_n - C$	16·8	17·4	17·9	18·3	19·0	19·8	21·7	24·0	25·4	

The magnitude of the nebula m_n is either photographic or visual; the magnitude m_* of the star is photographic. The constant C, according to the assumptions made, is the same for all nebulae and all temperatures. Of course its value when m_n is determined photographically is different from its value for visual m_n,s.

To evaluate C it is sufficient to have one good temperature determination from electron excitation (nebulium) for one nebula, when also m_* and m_n are known for this nebula. After this, the temperature scale is fixed, and the temperature T from electron excitation follows directly from the difference $m_* - m_n$.

* See Section 6.3.

6.2. *The stellar temperatures for a number of planetary nebulae by the method of Section 6.1.*—Photographic magnitudes of central stars in planetary nebulae have been estimated by Curtis[19]. In several instances Van Maanen or Hubble have determined photographic magnitudes, and by comparison with Curtis' values Hubble[20] obtains the correction $\triangle = -0.173m + 3.37$, to be added to Curtis' m. Table 14, column 2, contains Curtis' photographic magnitude of the central star corrected according to Hubble. When direct determinations by Hubble or Van Maanen were available, their value of m has been taken instead, and when there were determinations by both observers, the average of the two has been taken.

Visual magnitudes of planetary nebulae have been determined by Holetschek.[21] The comparison stars were either estimated by him, or the magnitudes of the B.D. were used. In several cases Hopmann[22] has made a photometric determination of the magnitudes of the comparison stars, and thus revised Holetschek's values. The values of the visual magnitudes of planetary nebulae according to Holetschek and Hopmann are given in column 3 of Table 14. Where such values were not available, it has been assumed that Holetschek's determinations were based on the B.D. scale and this scale has been reduced to Hopmann's photometric scale by the following correction given by Hopmann, which has been added to Holetschek's magnitudes.

Magn. nebula.	9.3	9.4	9.5	9.6	> 9.6
Correction.	0.5	0.7	0.9	1.1	1.1

Below 9.3 magnitudes the correction is not appreciable and has not been applied.

Holetschek and Hopmann's values refer to the combined visual magnitudes of star and nebula. For the present work the magnitude m_n of the nebular envelope is needed, but this is practically the same for bright nebulae. It has been considered that the nebula is bright enough for this, when $m_* - m_n$ is larger than 2. Nebulae for which this difference is 2 or smaller have not been included in the table.

Column 4 contains the difference $d = m_* - m_n$. The errors, both in Holetschek and Hopmann's determinations and in Curtis' estimates are rather considerable and the error in d may well be 1 magnitude, so that the last figure is really not significant.

For the nebula N.G.C. 6543, $m_* = 11.3$ (Van Maanen), $m_n = 8.1$ (Hopmann, comparison stars measured), hence $d = m_* - m_n = 3.2$. For this nebula the temperature by the nebulium method is $T = 37,000°$ by Table 11 of Section 5. This temperature gives $m_* - m_n - C = 13.6$ from the graph representing Table 12. Hence $C = 3.2 - 13.6 = -10.4$.

For the nebula N.G.C. 6572, $m_* = 10.5$ (average of Van Maanen's value 10.8 and Hubble's value 10.2), $m_n = 8.4$ (Hopmann, comparison stars measured), hence $d = 2.1$. The temperature from the nebulium method in the present work is $T = 38,000°$ (Table 11), leading to $d - C = 13.7$ from the graph representing Table 12. Hence $C = 2.1 - 13.7 = -11.6$.

The disagreement between the two values of C may in part be due to errors in the present work, but it is believed to be caused mainly by the inaccuracy in d. If the

[19] H. D. Curtis. Lick Observatory Publications 13, 57 (1918).
[20] E. P. Hubble. Ap. J. 56, 400 (1922) Table V and [19].
[21] J. Holetschek. Annalen der Wiener Sternwarte 20, 93, 114 (1907).
[22] J. Hopmann. A.N. 211, 474 (1921).

average value $C = -11.0$ is taken, Table 12 reduces to Table 13, which is only applicable to the observational material as given in Table 14.

TABLE 13.—DIFFERENCE IN MAGNITUDE AND TEMPERATURE

T/1000°	25	30	35	40	45	50	55	60	70	80	90
d	0.0	1.3	2.3	3.0	3.7	4.2	4.7	5.1	5.8	6.4	6.9
T/1000°	100	120	150	250	500	750					
d	7.3	8.0	8.8	10.7	13.0	14.4					

The last column of Table 14 contains the temperature in thousands of degrees based on the scale of Table 13. The temperature values must be considered as determined by the mechanism of electron excitation, and the scale has been fixed by the two nebulae marked by stars.

In judging the temperatures in Table 14 it should be kept in mind that the error in d may well be one magnitude, whereas there are also observational errors in the photometric work of the two nebulae used in fixing the scale. This results in a rather large error in T as is seen from Table 13. Up to 40,000° the temperatures have been given within 1000°, and for higher temperatures within 5000°, but the actual error is much larger than that.

TABLE 14.—APPROXIMATE STELLAR TEMPERATURES FOR A NUMBER OF PLANETARY NEBULAE FROM THE OBSERVED DIFFERENCE IN MAGNITUDE

m_* is the photographic magnitude of the star according to Curtis, Hubble, and Van Maanen, m_n the visual magnitude of the nebula according to Holetschek and Hopmann, and d their difference. The two nebulae on which the temperature scale of Table 13 is based are marked by a star. T is the temperature according to this scale.

N. G. C.	m_*	m_n	d	T/1000°
6445	19.0	10.4	8.6	140
1952	15.9	8.4	7.5	105
2438	16.7	9.8	6.9	90
650.1	16.7	9.9	6.8	90
6853	13.6	7.3	6.3	80
6818	14.9	8.8	6.1	75
6720	14.7	8.8	5.9	70
3587	14.3	9.4	4.9	55
7009	11.7	7.2	4.5	55
7662	12.7	8.4	4.3	50
6905	14.5	10.7	3.8	45
3242	10.8	7.1	3.7	45
6309	14.1	10.8	3.3	40
6210	11.7	8.5	3.2	40
*6543	11.3	8.1	3.2	40
4361	12.8	10.1	2.7	38
6826	10.8	8.4	2.4	36
*6572	10.5	8.4	2.1	34

As was remarked in Section 5.3, the temperatures from about 60,000° on must be considered as lower limits which the theoretical mechanism requires. The cases where d is smaller than 2 have not been included. When the visual magnitude of the nebular envelope is equal to the photographic magnitude of the star the resulting temperature is about 25,000° (Table 13). It is certain that such cases of a faint nebular envelope occur and the lowest temperatures of stars in planetary nebulae according to the theory are therefore about 25,000° or even lower. The lower temperatures in planetary nebulae, 25,000°–35,000°, according to the theory are therefore of the same order as those known for ordinary O stars from other methods. That thus the lower part of the temperature scale from luminosity data fits in with the upper part of the temperature scale for ordinary stars is a point in favour of the theoretical mechanism. From the lower temperatures upward the series of temperatures offered in Table 14 shows no gaps; temperatures seem to be represented all the way from 35,000° up to 100,000°, though the higher are only lower limits. The conclusion seems justified that the approximate scale of temperatures as offered in Table 14 is both a reasonable extension of the temperature scale for ordinary stars and for the scale based on nebular luminosity which for temperatures around 40,000° was checked more in detail in Section 5.

The nebula N.G.C. 7009 gives a temperature of roughly 55,000° which confirms the order of the direct determination from nebulium in Table 11, and is further evidence that the value from nebulium for this nebula is definitely lower than that from He_{II}, though the determinations are unsatisfactory.

Of interest among the nebulae of high temperature in Table 14 are the ring nebula in Lyra, N.G.C. 6720 with a minimum stellar temperature of 70,000°, and the Dumb-bell nebula N.G.C. 6853 with a minimum temperature of 80,000°.

There are two nebulae which by the present method yield minimum temperatures of 100,000° and higher. They are N.G.C. 6445, 140,000° and N.G.C. 1952, 105,000°. In the case of N.G.C. 6445 the high value of d may be due to partial obscuration of the star, since, as Dr. Hubble informed me, there is much obscuring matter in neighbouring regions. It appears therefore better to discard this case entirely. For the Crab nebula N.G.C. 1952, there is strong evidence that the central star, really consisting of two components, is an old nova observed about 900 years ago.* An error of one magnitude in d would lower the temperature by about 20,000°, but on the other hand, as explained in Section 5.3, the simplified assumptions are such as to make the higher temperatures come out too low. On the basis of the mechanism discussed it is therefore very likely that in the Crab nebula a stellar temperature of 100,000° is reached. There are two cases in Table 14 where the theoretical mechanism yields a minimum temperature of 90,000°.

The conclusion is therefore reached that the mechanism of nebular luminosity discussed requires a stellar temperature as high as 100,000° for some of the planetary nebulae. Before this, Bowen had concluded that such high temperatures occur. His conclusion however is based on the relative intensities of lines in the nebular spectrum as given by Wright.

* To this, Dr. Hubble called my attention. Compare Leaflet 14, January 10, 1928, of the Astronomical Society of the Pacific, by Dr. E. Hubble.

The observational material on which Table 14 is based is very imperfect. The standards N.G.C. 6543 and N.G.C. 6572 by which the scale is fixed require improvement. In his work on slitless spectra of planetary nebulae at the Lick Observatory, Mr. L. Berman, according to a private communication, determines integrated intensities by means of a registering microphotometer. It is contemplated to use his method of integrating the intensities also for the films of the two nebulae of the present work, and thus improve the standards. This re-determination has been carried out meanwhile and the result will be published elsewhere. It appears of still more importance that accurate determinations are carried out of the difference in magnitude of nebula and star for a number of nebulae, preferably by photographic methods. Work of this kind is now in progress at the Observatory in Bergedorf. To such improved material it appears that the general method as given in Section 6.1, including Table 12, is applicable without change.

6.3. *The isophotic wavelength for O stars according to A. Brill.*[23]—The isophotic wavelength λ_{ph} of the photographic brightness for the temperature T is defined as that wavelength for which the spectral energy per wavelength unit of a black body of temperature T, apart from a constant which is independent of the temperature, is equal to the photographic surface brightness of the star at the Zenith.

The radiation temperature of the photographic brightness is defined as the temperature of a black body which, situated outside the atmosphere, would produce the photographic surface brightness of the star at the Zenith.

The following table is due to a private communication of Professor A. Brill, from calculations as yet unpublished.* The temperature T in the first row contains the values c_2/T, where c_2 is defined by $x = \dfrac{h\nu}{kT} = \dfrac{c_2}{\lambda T}$, and λ in μ, hence

$c_2 = 1.432 \times 10^4$ (compare 14a). The third row contains the isophotic wavelength in A in Brill's fundamental photometric system. The last row contains the change in the spectral energy, if the isophotic wavelength λ_{ph} is changed by 10 Å. If λ is increased, the spectral energy for the temperatures given decreases, hence the change is to be added to the spectral energy in magnitudes if λ_{ph} increases. The values up to $c_2/T=0.603$ were given by Professor Brill. Since for these values the relation between λ_{ph} and c_2/T is linear, the values for higher T's at Professor Brill's suggestion, have been obtained by linear extrapolation.

$T/1000°$	6.9	7.9	9.0	11.8	16.3	23.8	35.6	60.0	100
c_2/T	2.080	1.816	1.594	1.216	0.880	0.603	0.402	0.239	0.143
λ_{ph}^R	4306	4292	4280	4258	4238	4222	4210	4200	4195
	0.000^m	0.002^m	0.003^m	0.005^m	0.007^m	0.008^m	0.009^m	0.009^m	0.010^m

If the accuracy desired is only 0.1^m, the table shows that for stellar temperatures between 23,800° and 100,000° one may safely assume the constant value of 4210 Å for the isophotic wavelength, since this introduces an error of the order 0.01^m. This isophotic wavelength holds in Brill's fundamental photometric system. In other photometric systems the isophotic wavelength is somewhat different. Professor Brill, according to private communi-

[23] A. Brill. Veröffentlichungen der Universitätssternwarte zu Berlin-Babelsberg, Band V, Heft 1 (1924).
*This work is now published in Veröffentlichungen der Universitätssternwarte zu Berlin-Babelsberg, Band VII, Heft 5 (1929). in particular tabelle 4.

munication, has calculated that e.g. the isophotic wavelength for King's photographic magnitudes (Harvard Annals 76,6) is smaller by 25 Å, for Hertzsprung's photographic magnitudes (B.A.N. 35 201) by 51 Å than in the fundamental system. If one would take for King's or Hertzsprung's photographic magnitudes the value 4210 Å, in the temperature region from 23,800° on, this would introduce an error smaller than $0 \cdot 1^m$.

Though strictly speaking each photometric system would have to be considered separately, it would appear that, if no higher accuracy than $0 \cdot 1^m$ or $0 \cdot 2^m$ is required, one is justified to take λ_{ph} for O stars equal to 4210 Å. Because of the very approximate nature of the temperature determination considered in Section 6·1, a higher accuracy than about $0 \cdot 2^m$ does not appear to be required, and therefore the results of section 6.1 based on $\lambda_{ph} = 4210$ Å for O stars appear to be applicable also for future work.

It should be pointed out that in the discussion of the isophotic wavelength just given, the spectral intensity is expressed per unit wavelength, in accordance with Professor Brill's definition. In the derivation of the formulae for the temperature determination, the spectral intensity per unit frequency is used. This is only permitted when the isophotic wavelength for the region considered is constant. This, however, is justified for O stars if no higher accuracy than $0 \cdot 2^m$ is required, since the errors when λ_{ph} refers to the spectral intensity per frequency unit are of the same order as when it is referred to the spectral intensity per wavelength unit.

We are indebted to Professor Brill* for placing the data at our disposal prior to publication and explaining the way in which they can be applied to higher temperatures and how the discrepancy due to a change in λ_{ph} is obtained.

SECTION 7.—CONSIDERATION OF SOME DETAILS. DIFFICULTIES IN THE THEORY. CONCLUSION.

7.1. *The theoretical occurrence of secondary processes in the emission of the Lyman series.*—The formula (5) of Section 4.2 was derived by the author[24] under the assumption that practically all quanta of the Lyman series leave the nebula as the first line of the series Ly_α. When the Lyman series is first produced by recombination one may expect that all the energy is practically concentrated in the first few lines, say from Ly_α up to Ly_ϵ, and it is sufficient to consider these lines only. After this first re-emission secondary processes may take place by repeated absorption and re-emission of the lines of the Lyman series. Thus one quantum Ly_β, on its way out of the nebula, may be absorbed by a hydrogen atom and re-emitted as $Ly_\alpha + H_\alpha$. In general a quantum in any of the lines Ly_β to Ly_ϵ will be split up into one quantum Ly_α and other quanta, provided that these secondary absorptions and re-emissions take place a sufficient number of times, that is, provided that the absorption coefficient in one single spectral line Ly_β to Ly_ϵ is sufficiently large as compared with the absorption coefficient of the continuous spectrum at the head originally absorbed.**

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* Professor Brill, however, has pointed out privately that some caution is necessary, particularly when the plate used has an abnormal sensitivity curve. It may also be said that for O stars the atmospheric extinction in the region of shorter wavelength should be investigated more closely.

24 See 11.

** Since the absorption of the Ly_ϵ has been assumed to be practically complete, the "mean free path" of a quantum Ly_ϵ is comparable with the dimensions of the nebula or smaller. When the absorption coefficient for one of the Ly_β lines is much larger than for Ly_ϵ, the mean free path of such an Ly_β quantum is very small, and many absorptions and re-emissions may occur.

In the following the absorption coefficient of a line of the Lyman series under conditions presumably prevailing in nebulae will be calculated and compared with the absorption coefficient of the continuous spectrum at its head. As is well known, this absorption coefficient may be obtained from the number of dispersion electrons in the following manner:—

The energy scattered per unit time by one classical space-oscillator is given by the expression

$$Z = \frac{\pi \epsilon^2}{m} \rho_\nu \qquad (18),$$

where ϵ and m are the electronic charge and mass, and ρ_ν the energy density per frequency unit of the incident radiation.

Let the atoms have velocities v; then due to the Doppler effect one may assign a number of normal atoms dN per cm.³ within a frequency interval dv which are capable of absorbing the incident radiation, ν and v being related by $(\nu - \nu_r)/\nu_r = v/c$, where ν_r is the frequency of an atom at rest. This Doppler effect seems the only reasonable cause by which the widening of nebular lines may be produced.

Let f be the number of dispersion electrons assigned to the absorption of a certain line by one normal atom. Then there are $f \, dN$ oscillators per cm.³ capable of absorbing radiation within dv. If the radiation travels a distance dx through a cross-section of 1 cm.² the energy absorbed per unit time is therefore, according to (18)

$$f \frac{\pi \epsilon^2}{m} \rho_\nu \, dN \, dx.$$

The energy incident per unit time per cm.² within dv is $c \, \rho_\nu \, dv$, and hence the absorption coefficient in a line

$$\alpha_l = f \frac{\pi \epsilon^2}{m c} \frac{dN}{c \, dv} \qquad (19).$$

If $\Delta \nu$ is the width of the spectral line, and the Doppler effect that of an irregular motion, e.g. Maxwell distribution, one may put, as regards order of magnitude

$$\frac{dN}{dv} \sim \frac{N}{\Delta v}$$

N being the number of normal atoms per cm.³ The absorption coefficient per atom of a line is therefore of the order

$$\alpha_l = f \frac{\pi \epsilon^2}{m c} \frac{1}{v} \left(\frac{v}{\Delta v} \right) \qquad (20).$$

For the absorption of the continuous spectrum beyond the head of the Lyman series let the number of dispersion electrons within the frequency interval dv be df. The absorption coefficient per atom of the continuous spectrum is then given by

$$\alpha_c = \frac{\pi \epsilon^2}{m c} \frac{df}{dv} = \frac{\pi \epsilon^2}{m c} \frac{1}{R} \frac{df}{dE} \qquad (21),$$

obtained in a way similar to (19). In this expression $E = \frac{\nu}{R} - 1$, R the Rydberg constant.

For this continuous spectrum an expression for df/dE was derived by Sugiura[25] from the wave mechanics. His expression (15) gives e.g. for the absorption at the head of the Lyman series $df/dE = 0.782$, and hence from (21) the absorption coefficient of the continuous spectrum becomes

$$\alpha_c = 0.54 \times 10^{-17}$$

Table 1 of Sugiura's paper gives the value of f for the lines of the Lyman series. For the fifth line Ly_e, $f = 0.0078$. Since $\nu = 3.19 \times 10^{15}$ and $\pi e^2/mc = 0.0264$, the approximate formula (20) gives for the absorption coefficient per atom for Ly_e

$$\alpha_e = 0.65 \times 10^{-19} \left(\frac{\nu}{\Delta \nu}\right)$$

The atomic absorption coefficient for Ly_e is therefore of the same order as the atomic absorption coefficient α_c at the head of the series, if $\nu'/\Delta\nu$ is of the order 100.

For the Orion nebula Buisson, Fabry, and Bourget's interference measurements[26] give a width of the line H_γ corresponding to an irregular Doppler motion of the order 10 km./sec.; hence $\nu/\Delta\nu$ is of order 3×10^3 for hydrogen. In this case the absorption coefficient for Ly_e is therefore of the order 3,000 times that of the continuous spectrum Ly_e. Though for planetaries the irregular Doppler effect may be much larger, the conclusion seems justified, that in general the absorption coefficient of one of the first lines in the Lyman series is much larger than that of the continuous spectrum at its head for conditions prevailing in nebulae, and that the assumption of secondary processes on which formula (5) is based is theoretically justified.

7.2. *Evaluation of integrals by Debye's method.*—The integrals I_1 and I_2 in Table 7 were evaluated according to a method due to Debye. The method of evaluation for I_2 is given in Debye's paper on the specific heat of solids,[27] and the method for I_1 is quite analogous. The formulae used for the evaluation of the two integrals are as follows.

Small values of x_o (0 to 2). For small values of x the following expansion by means of Bernoulli numbers is used, which is convergent for $x<2\pi$.

This gives:

$$\int_0^{x_o} \frac{x^{n+1}}{e^x - 1}\, dx = \frac{x_o^{n+1}}{n+1} - \frac{x_o^{n+2}}{2(n+2)} + B_1 \frac{x_o^{n+3}}{2!\,(n+3)} - B_2 \frac{x_o^{n+5}}{4!\,(n+5)} + B_3 \frac{x_o^{n+7}}{6!\,(n+7)} + \cdots$$
$$- B_4 \frac{x_o^{n+9}}{8!\,(n+9)} + \cdots$$

Substituting the values of the Bernoulli numbers: $B_1 = 1/6$, $B_2 = 1/30$, $B_3 = 1/42$, $B_4 = 1/30$, this gives for $n=1$

$$\int_0^{x_o} \frac{x^2}{e^x - 1}\, dx = \frac{x_o^2}{2} - \frac{x_o^3}{6} + \frac{x_o^4}{48} - \frac{x_o^6}{4320} + \frac{x_o^8}{241,920} - \frac{x_o^{10}}{12,096,000} + \cdots \quad (21).$$

[25] M. Y. Sugiura. Journal de Physique et Le Radium, Série VI, Tome VIII, 113 (1927).
[26] H. Buisson, Ch. Fabry, and H. Bourget. Ap. J. 40, 241 (1914).
[27] P. Debye. Annalen der Physik 39, 798, 799 (1912).

The integral between the limits 0 and ∞ is $2! \sum_1^\infty \frac{1}{n^3} = 2.40411$, and subtracting from this the values from (21) gives I_1 as contained in the second column of Table 7 from $x_o = 0$ to 2.

Similarly, for $n=2$, one obtains

$$\int_0^{x_o} \frac{x^3}{e^x - 1}\, dx = \frac{x_o^3}{3} - \frac{x_o^4}{8} + \frac{x_o^5}{60} - \frac{x_o^7}{5040} + \frac{x_o^9}{272,160} - \frac{x_o^{11}}{13,305,600} + \cdots \quad (22).$$

This integral between the limits 0 and ∞ is $\frac{\pi^4}{15} = 6.4939$, and subtracting from this the values from (22) gives I_2 as contained in the third column of Table 7 from $x_o=0$ to 2.

Large values of x_o (2 to ∞). For large values of x, the expansion

$$\frac{1}{e^x - 1} = \sum_{n=1}^\infty e^{-nx}$$

is used. By partial integration for each value of n under the summation sign, one obtains

$$I_1 = \int_{x_o}^\infty \frac{x^2}{e^x - 1}\, dx = \sum_{n=1}^\infty e^{-nx_o} \left(\frac{2}{n^3} + \frac{2\,x_o}{n^2} + \frac{x_o^2}{n}\right) \quad (23),$$

$$I_2 = \int_{x_o}^\infty \frac{x^3}{e^x - 1}\, dx = \sum_{n=1}^\infty e^{-nx_o} \left(\frac{6}{n^4} + \frac{6\,x_o}{n^3} + \frac{3\,x_o^2}{n^2} + \frac{x_o^3}{n}\right) \quad (24).$$

For $x_o = 2$ and higher the values of I_1 and I_2 as given in Table 7 were obtained from (23) and (24). For the number of decimal places given, it suffices to retain the first five terms in the expansion ($n = 1$ to 5), and as x_o becomes larger, more terms may be dropped.

For $x_o = 2$, I_1 and I_2 were obtained both by the method for large x_o and for small x_o.*

7.3. *Difficulties in the theory. Conclusion.*—The theory discussed is only intended to be applicable to the luminosity of diffuse and planetary nebulae, the latter being considered more particularly in the present paper.

The *main assumptions* are that the nebulae are excited to luminosity by far ultra-violet starlight. This ultraviolet starlight produces photo-electrons, and the luminosity is assumed to be due to two mechanisms:

1. Recombination, which is the main source of the spectra of H, He_1, He_{11}, etc.
2. Electron-excitation by photo-electrons prior to recombination (Bowen), which is the main source of the luminosity of lines of low excitation, in particular the resonance "nebulium" lines, which all happen to originate from metastable states, though this is not a requirement for the mechanism.

* On a previous occasion (Ap. J. 65, 50, 1927), the integral I_1 was obtained by quadrature. There the accuracy being 2 per cent for the first two intervals and 1 per cent for the subsequent intervals, the accuracy does not correspond to the number of places given. In the present evaluation by Debye's method the integrals are accurate to the number of places given.

In order to make a quantitative treatment possible, a number of subsidiary assumptions have been introduced, which are all of an approximate nature. These, though some of them seem *a priori* reasonable, have their main justification in the fact that they correlate the observational data dealt with in the present paper and apparently place them in a consistent scheme. Such *subsidiary assumptions* are, mainly

1. That the distribution of energy in the spectrum of the exciting star is that of a black body, even into the far ultraviolet. It is realized that this is only an approximation and that in particular absorptions in the ultraviolet may occur, but the experience that, in the visual and photographic regions, stars approximately behave like black bodies modified by absorption, makes the assumption a reasonable one.

2. That the nebula is in a stationary state.

3. That the absorption of ultraviolet starlight, in particular by hydrogen, is practically complete.

4. That for the recombinations, in particular for hydrogen, the number of recombinations on the first and second level, producing Ly_c and Ba_c is neglected with respect to the recombinations on higher levels.

5. That an appreciable portion of the energy of the free electrons before they recombine is given up in the excitation of the nebulium resonance lines, and that of these again an appreciable portion of the energy is in the nebulium lines measured, in particular N_1 and N_2.

The agreement of temperature derived under the subsidiary assumptions from the luminosity by different methods is considered as a justification of the assumptions made, though the test has been carried out mainly for temperatures around 40,000°.

The theory discussed, even as far as its main assumptions go, does not propose to explain the excitation of all bright lines occurring in celestial objects. In particular no attempt is made to explain the occurrence of bright lines in early type stars and other types of variables.

As far as its application to nebulae goes, a breakdown of some of the subsidiary assumptions in certain cases, e.g., for very high temperatures, must be kept open as a possibility.

From observational evidence, *difficulties* in the theory discussed here have been forwarded by H. H. Plaskett[28]. In the case of Z Andromedae the observational evidence results in a low temperature of the associated star, the Z star, about 6,000°. This star is of $9 \cdot 6^{m}$, is variable in an irregular manner by about $0 \cdot 4$ magnitude, and has emission lines, principally of H, He_{I}, Mg_{II}, and Fe_{II}. The observed luminosity of the Z nebula is about 100^{m} stronger than would follow from a star of the given magnitude and this temperature according to the present theory. This presents a serious difficulty for the theory, if the Z star is looked upon as the exciting star. It is true that the star is not of the same type as occurring in normal planetaries, and also its variability might be connected

28 H. H. Plaskett. Publications of the D. A. O. IV,119 (1928).

with phenomena as yet not understood. On the other hand, the Z nebula shows many of the characteristics of the envelope of an ordinary planetary nebula. Plaskett points out that the luminosity of the nebula might be produced by a star of about 15^{m} on the basis of the theory of luminosity, which through its faintness would then not be observable, in which case according to him the Z nebula and Z star would have to be supposed as only optically coincident. Dr. Menzel, however, in work yet unpublished, has forwarded as a possible explanation that the star in Z Andromedae is a physical binary consisting of the Z star as primary and an O star of about 15^{m} as its companion, so that the case would be somewhat analogous to Mira Ceti as investigated by Joy[29], where the existence of a later type bright line star with a companion of early type has been established. A still more striking case is the Mira-variable R Aquarii with companion, the latter being the nucleus of a planetary nebula. P. W. Merrill. Pub. A.S.P., 39, 48 (1927). Ap. J. 65, 293 (1927). To this case Dr. Baade called my attention. We are indebted to Dr. Menzel for permitting us to quote this consideration, which appears to offer a way out of the theoretical difficulty, in advance of publication.

As regards diffuse nebulae, Plaskett points to the discrepancy which there appears to be with the theory for the North America Nebula, N.G.C. 7000. The star α Cygni, which may be associated with it (Hubble), has, according to Plaskett, a temperature of less than 10,000°, which would be inadequate to produce nebular emission lines. Thus this case would neither agree with Hubble's empirical rule that a star of normal type producing emission lines should be B_1 or earlier, nor with the present theory. On the other hand, Hubble has concluded that "the enhanced α Cygni type of stellar spectrum is so unique that unusual properties may be suggested without prejudice to the existing data."[30] Another difficulty for the theory raised by Plaskett is that of nine diffuse nebulae excited by O type stars only one shows He_{II} λ 4686, whereas of five nebulae excited by B type stars three show this line. This difficulty may be left as it stands, and requires further investigation.

Conclusion.—It is realized that in the present work a number of assumptions have been introduced which by their very nature must be approximate. It has been shown that for the quantitative material on planetary nebulae of the present paper, secured from slitless spectrograms, these assumptions lead to a consistent, even though only approximate, picture, as is shown from the agreement between the temperatures obtained by different methods from luminosity data only. Though it is realized that in extreme or abnormal cases some of the subsidiary assumptions may break down, the conclusion seems justified that the assumption that the exciting star behaves approximately like a black body and that the luminosity is produced by known physical mechanisms gives a fair insight into the facts.

The author wishes to express his sincere thanks to the Director, Dr. J. S. Plaskett, for having had the privilege of working at the Dominion Astrophysical Observatory and carrying out the present investigation, and for help in the observational problems. The guidance of the Director and the members of the Staff for the observational work has been much appreciated.

29 A. H. Joy. Ap. J. 63, 281 (1926).
30 See 4.

THE ORBIT OF H.D. 32990 (103 TAURI, BOSS 1216)

BY S. N. HILL

ABSTRACT

Ninety-five spectra of H.D. 32990 are the basis of the determination of the spectroscopic orbit. Fifty-nine of these observations were taken at the Mt. Wilson Observatory and combined with the thirty-six Victoria observations, resulting in a period of 58·31 days, eccentricity 0·189 ±0·017, semi-amplitude 36·73 ±0·66 km. and $\gamma+16·22 ±0·48$ km. The probable error of a plate is ±4·46 km. per second.

The star H.D. 32990 (103 Tauri, Boss 1216) 1900 $\alpha = 5^h 2^m \cdot 0$, $\delta = +24° 08'$, visual magnitude 5·50 and type B3 was announced as a spectroscopic binary by Adams, Joy, and Sanford[1], with a range of -26 to $+50$ km. from 18 plates. Three observations of the star made at the Yerkes Observatory were published by Struve.[2] A number of spectra of this star were secured during the investigation of B type stars at Victoria. The lines were fair, and since the second and third plates showed a range from $-11·9$ to $+60·9$ km., it was considered advisable to obtain more spectra with a view of determining the orbit.

It was only after all the spectra were obtained and when enquiry was made for more definite particulars of some of the published observations, it developed that Dr. Sanford, of Mt. Wilson Observatory, had already secured upwards of 50 spectra of this star. Considerable correspondence over this binary passed between the two observatories with intercommunication of observations and suggestions as to period. Dr. Sanford suggested possible periods of about 60 or 20 days without feeling satisfied of their correctness and the writer suggested a period of less than a day, which was also not satisfactory. Indeed, this short period was definitely ruled out by the Mt. Wilson observations on J.D. 5220 where over an interval of 0·22 days there was a total range of only 5 km./sec.

Owing to peculiarities in some of the lines of the spectra and uncertainty as to velocities, it was felt desirable to remeasure the Victoria plates. These remeasures resulted in some material changes in the velocity of several of the spectra and when these new velocities were plotted it was almost immediately seen that a period of about 58 days fitted the observations approximately.

[1] Pub. A.S.P. 36, 137, 1924.
[2] Ap. J. 64, 12, 1926.